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MILL-GEARING.

A

PRACTICAL TREATISE

ON

MILL-GEARING,

WHEELS, SHAFTS, RIGGERS. ETC.,

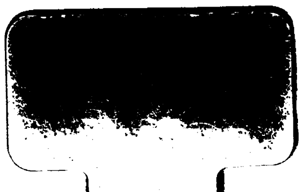
FOR

THE USE OF ENGINEERS.

By THOMAS BOX,

…TICAL HYDRAULICS,' 'TREATISE ON HEAT,' ETC.

… CROSS.



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PREFACE.

The circumstances which have led to the preparation of the following work may be briefly explained. The author was extensively engaged for many years in the practical construction of Mill-gearing; he found the existing rules for calculating the Power of Wheels and Shafts very unsatisfactory; and as to Riggers, the only known rules are of comparatively modern date, and are given in such a form as to be almost useless for practical purposes. Under these circumstances, he was led to search for Rules and to construct Tables for his own daily use, and he now gives the results to the world. It is, perhaps, necessary to say thus much, because the rules are for the most part novel, and it may be well to explain that they have not been adopted in pursuance of any favourite theory, but are the result of the inflexible teachings of a long and varied experience. For the same reason, many examples are given of Wheels, Shafts, Riggers, &c., in practice, that the reader may see how far the rules agree therewith; in fact about one-third of the Tables in the work are devoted to that special purpose. As many of the Rules were empirical, and did not admit of a theoretical demonstration, the only course open for proving their correctness, was by giving examples of successful application.

BATH, *March*, 1869.

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PRACTICAL TREATISE ON MILL-GEARING.

CHAPTER I.

ON THE STANDARD UNIT OF POWER, &c.

(1.) "*Unit of Power.*"—The standard unit of power in this country is the "foot-pound," or the power necessary to raise 1 lb. avoirdupois 1 foot high per minute; but among practical engineers the unit most commonly used is the "horse-power," being the power necessary to raise 33000 lbs. 1 foot high per minute, or, in other words, 33000 foot-pounds per minute.

This is James Watt's old standard, and its application appears very simple; but certain allowances have to be made for the friction of the engine itself, and of the pumps or other machinery by which the work has to be done, and complications arise which have led to the use of such terms as "nominal horse-power," "indicated horse-power," "nett horse-power," "gross horse-power," &c., &c., which are very confusing, especially to non-professional men. We shall have to use principally the term "nominal horse-power" in this work; and it may be well at the outset to explain what we understand by that term, and its relation to the others we have named. Perhaps this can be done best by examples.

(2.) "*Nominal Horse-power,*" *&c.*—We will take the case of a steam-engine raising 200 gallons of water per minute 330 feet high, by a set of deep-well pumps. The weight of a gallon of water being 10 lbs., this is equal to $\frac{2000 \times 330}{33000} = 20$ horse-power; and to do this work we should employ an engine of 20 *nominal* horse-power. But let us suppose that the friction of the pumps, &c., was such that they gave out in useful work

only ·66 or ⅔rds of the power absorbed by them, and this is about the fact, as shown by Table 1; they would therefore require

TABLE 1.—Of the MODULUS of MACHINES for RAISING WATER, being the ratio of the work done to the nett indicated Horse-power consumed.

	Modulus.
Inclined chain-pump	·38 to ·40
Upright ditto	·53 ,, ·65
Archimedean screw	·62 ,, ·70
Deep well and mine pumps	·66
Persian wheel, with swinging buckets	·60
Chinese wheel, with oblique-fixed buckets	·58
Endless chain and buckets, lift 3 ft. 3 in. to 6 ft. 6 in. ..	·48
Ditto ditto 8 ,, 2 ,, ,, 8 ,, 6 ,, ..	·57
Ditto ditto 9 ,, 10 ,, ,, 10 ,, 10 ,, ..	·63
Ditto ditto 11 ,, 9 ,, ,, 13 ,, 0 ,, ..	·66
Centrifugal pump, 5 feet lift, large size, 10,000 gallons ..	·75
Ditto 5 ditto small 1,400 ditto ..	·65
Ditto 30 ditto large 1,400 ditto ..	·55
Ditto 30 ditto medium 400 ditto ..	·45
Ditto 30 ditto small 100 ditto ..	·25

$\frac{20}{\cdot 66}$ = 30 horse-power, to do the work; and if an indicator were placed on the cylinder, it would have to show 30 horse-power *indicated*, clear of the friction of the engine itself. To complete the illustration, say that the friction of the engine itself was equal to 10 horse-power by the indicator, and we should then have 20 *nominal*, 30 *nett indicated*, and 40 *gross indicated* horse-power. It appears, then, that in this case the nett indicated horse-power is 50 per cent. more than the nominal, and the gross indicated power is double the nominal; and this may be taken as a fair average, according to the practice of our leading engineers. It will be observed that in this case the nominal horse-power is identical in amount with the real work done, and the friction of the pumps, &c., is compensated by the excess of the indicated power over the nominal. But this will not hold true in every case, and with all kinds of machinery; in fact, it will apply without correction only to those cases where the machines give out ·66 of the power consumed by them;

and Table 1 shows that this is the modulus of the best class of upright chain-pumps, Archimedean screw, deep-well pumps, endless chain of buckets for lifts over 13 feet, and centrifugal pumps for low lifts under 5 feet, &c., &c.

With other machinery, having a modulus of friction more or less than ·66, the nominal horse-power of the engine will not agree exactly with the work to be done. For instance, if we had to raise 1400 gallons 33 feet high per minute, by a centrifugal pump whose modulus by Table 1 is ·55, we should require $\frac{14000 \times 33}{33000 \times \cdot 55} = 25 \cdot 5$ nett indicated horse-power, which by the ratio we have given would be equal to an engine of $\frac{25 \cdot 5}{1 \cdot 5} = 17$ nominal horse-power, and the wheels, shafts, &c., should also be calculated for that power by the rules and tables in this work. Again, to raise 100 gallons per minute 50 feet high, by a small centrifugal pump, whose modulus, by Table 1 is ·25, we should require $\frac{1000 \times 50}{33000 \times \cdot 25} = 6$ nett indicated, and an engine of $\frac{6}{1 \cdot 5} = 4$ nominal horse-power.

(3.) It is much to be desired that the term nominal horse-power should be abandoned altogether, and indicated horse-power universally substituted, or otherwise that there should be some fixed ratio established by authority between the two; at present it is quite an indefinite term. We have just fixed the ratio of the nominal to the gross indicated power at 1 to 2, and this agrees with the practice of most engineers, especially for land-engines, but there is no fixed rule; thus we hear of marine-engines of 1200 nominal horse-power, which are intended to work up to six times! their nominal power, and which on trial actually do show 8000 gross indicated horse-power, or nearly 7 times the nominal.

Before calculating the sizes of wheels, shafts, &c., by the rules in this work, the real or indicated power to be dealt with should be ascertained or carefully estimated, and the nominal horse-power deduced from it, allowing the ratios of the "nominal;"

the "nett indicated" and the "gross indicated" power to be 1, $1\frac{1}{2}$, and 2. The ratio $1\frac{1}{2}$ to 2 is adopted for the sake of round numbers, it gives, perhaps, too great an allowance for friction with large engines; $1\frac{1}{2}$ to $1\frac{3}{4}$ would be nearer the truth for such cases. In the case just given, the gross indicated power being 8000, the nett indicated would be 6000, and the nominal 4000 horse-power; the size of shaft, &c., may be calculated for the latter by the rules in (51), and in the Appendix. A serious error would have ensued in this case by using the reputed nominal power of 1200 horses in fixing the size of shaft.

We have used the term nominal horse-power in this work in deference to custom; to have substituted indicated horse-power, would have been to render the book useless, or nearly so, to most practical men, by whom the old term is almost universally employed. By reducing the indicated to the nominal power in the way we have illustrated, we may obtain the precision of the former with the convenience of the latter.

(4.) "*Power of Horses, Men,*" *&c.* The power of animals varies greatly with the mode in which it is exerted, and the duration of the labour. In the case of a horse, the most favourable mode of exerting the strength, is in drawing a carriage at a low velocity, but even then it is not equal to the standard unit of 33000 foot-pounds per minute, except when the duration of the labour is reduced to 6 hours per day. When a horse walks in a circle for the purpose of driving machinery, his effective power is reduced considerably by the unfavourable rotary motion; the diameter of the circular path should not be less than 20 or 25 feet, and the velocity should not exceed $1\frac{3}{4}$ to 2 miles per hour. When the power is employed in raising water by pumps the useful effect is still further reduced by friction. Table 2 gives the power of men, horses, and other animals, all reduced to the standard of horse-power. The following examples will illustrate its application to cases in practice:—

Say we had to raise 20 gallons per minute 120 feet high, for 8 hours per day, by a set of deep-well pumps:—the work done is $\frac{200 \times 120}{33{,}000} = \cdot 73$ horse-power nett, and will require by Table 2,

TABLE 2.—Of the POWER of HORSES, MEN, &c., &c.

	Velocity in feet per Minute.	Duration of Labour in hours per Day.							
		2	3	4	5	6	8	10	12
		Nett Horse-power, Work done.							
Horse, walking in a circle	176	1·06	·867	·751	·672	·614	·531	·475	·434
Pony or Mule ditto	180	·708	·578	·501	·448	·409	·354	·315	·289
Bullock ditto	120	..	..	·668	·598	·545	·472	·422	·386
Ass ditto	157	..	..	·208	·186	·170	·147	·131	·120
Horse, in circle, raising water by deep-well pump ..	176	·708	·578	·501	·448	·409	·354	·315	·289
Pony or Mule ditto ditto ..	180	·473	·386	·334	·300	·272	·236	·211	·190
Bullock ditto ditto ..	120	..	..	·445	·398	·364	·315	·281	·257
Ass ditto ditto ..	157	..	..	·138	·124	·113	·098	·088	·080
Man working at a winch	220	·160	·131	·113	·101	·092	·080	·072	·065
Ditto tread-wheel	30	·236	·193	·167	·149	·136	·118	·105	·096
Ditto capstan	118	·189	·154	·134	·120	·109	·094	·084	·077
Ditto with winch and deep-well pump ..	220	·107	·087	·075	·067	·061	·053	·048	·043
Ditto tread-wheel ditto ..	30	·157	·128	·112	·098	·091	·079	·070	·064
Ditto capstan ditto ..	118	·125	·102	·088	·079	·072	·062	·055	·051

$\frac{\cdot 73}{\cdot 354} = 2$ horses, or $\frac{\cdot 73}{\cdot 053} = 14$ men working the pumps by winches.

Again, with an upright chain-pump, say we have to raise 200 gallons per minute 20 feet high, for 6 hours per day:—the nett work done is $\frac{2000 \times 20}{33{,}000} = 1 \cdot 21$ horse-power; and taking the modulus of the pump from Table 1, at $\cdot 53$, this is equal to $\frac{1 \cdot 21}{\cdot 53} = 2 \cdot 3$ horse-power indicated, and we shall require $\frac{2 \cdot 3}{\cdot 614} = 4$ horses, or $\frac{2 \cdot 3}{\cdot 109} = 22$ men working the pump by a capstan. Here it will be observed that we take the nett power of the horse, or the man, from Table 2, divested of the friction of pumps, &c., which is allowed for in the modulus $\cdot 53$.

Again, say we have to fix the size of a set of deep-well pumps to be worked by a donkey, the lift being 100 feet, and the duration of the labour 5 hours per day. Then from Table 2, the nett work of a donkey with pumps is $\cdot 124$ horse-power, or $\cdot 124 \times 33{,}000 = 4092$ foot-pounds, or $\frac{4092}{100} = 41$ lbs., or say 4 gallons per minute, raised 100 feet. A horse would have raised $\frac{\cdot 448 \times 33{,}000}{100 \times 10} = 14 \cdot 8$ gallons, and a man with a winch-pump $\frac{\cdot 067 \times 33{,}000}{100 \times 10} = 2 \cdot 2$ gallons per minute, &c., &c.

CHAPTER II.

ON WHEELS—FORM OF TEETH.

(5.) "*Pitch.*"—The pitch of a wheel is the distance from centre to centre of two contiguous teeth; but there are two ways in which this may be measured, namely along the curved pitch-line,—and in a direct straight line from centre to centre, or

in other words, along the *arc*,—and along the *chord*, and there is a great difference of opinion as to which of the two is correct. If this question were a mere distinction or definition of terms, it would be an unimportant matter, but in the case of a small pinion working with a large wheel or with a rack, a considerable positive error ensues from the adoption of an erroneous principle, and the wheels will not work together correctly. Professor Wallace, Donkin, Templeton and others have adopted the chord or straight line mode of measurement, and have given tables for calculating the diameter of wheels on that principle. It is a result of the chord principle, that the diameters of wheels are not exactly proportional to the number of teeth: for instance, a wheel with ten teeth is not precisely half the diameter of one with twenty teeth, &c., &c. Professor Willis, Grier, and others measure the pitch by the arc or along the curved pitch-line, and this is unquestionably the correct mode. Perhaps an *ocular* demonstration will be the most convincing, especially to practical men: let Fig. 1 represent to a scale half the full size, a pinion of ten teeth, working into a rack, the pitch being four inches, and to illustrate the matter more clearly let there be *no clearance* between the teeth; for the same reason the teeth are drawn of extra length. The form of the teeth having been accurately determined by rolling circles as explained in (7), the correctness of the principles (whatever they may be) on which the pitch is measured is manifested by the accuracy with which the teeth fit into one another, being in contact at the points A.B.C.D.E.F. If now the direct distance G.P.M be taken accurately in the compasses and compared with the pitch of the rack, it will be found to be about $\frac{1}{15}$th of an inch less than Q.R; but if the pitch of the pinion be measured along the curved line by short steps G.H.I.J.K.L.M, and the same distances be set off from Q, on the pitch-line of the rack at H′. I′. J′. K′. L′. R, the two sets of measurements will exactly coincide, and thus the method of measuring the pitch along the curved pitch-line is proved to be correct. To make the error of the other, or chord principle more palpable, let the direct distance G.P.M. be set off three times from R to S. T and U, and it will be found that

by the accumulation of the error, the centre of the tooth U is about $\frac{2}{10}$ths of an inch wrong in position, and that the teeth would cut into one another to that extent at A . W as shown by the dotted line V . W . Z, and all the rest would do the same, only to a less extent.

(6.) "*Diameter of Wheels.*"—The primary idea of a pair of wheels is the case of two plain cylinders working against each other and driving by frictional contact; these are called the *pitch-circles* and by the diameter of a wheel, the diameter of these imaginary circles is always understood. The diameter of a wheel with a given pitch and number of teeth, may be found from any table of circumferences of circles, or multiplying the pitch by the number of teeth, and dividing the product by 3·1416 will give the diameter in inches; thus a wheel with 122 teeth 2 inches pitch, must be $\frac{122 \times 2}{3 \cdot 1416} = 77 \cdot 66$ inches diameter at the pitch-line. The diameter may also be calculated by the constant multipliers in col. 2 of Table 3; thus in our case we have $\cdot 6366 \times 122 = 77 \cdot 66$ inches as before.

(7.) "*Form of Teeth.*"—The form of teeth should be such as to give the same perfect uniformity of motion as with plain cylinders rolling on one another. Mathematicians have shown that there are two curves which fulfil that condition, namely the *epicycloid* and the *involute.*

The Epicycloid is a curve drawn by a point in the circumference of a circle rolling on another circle, or on a plane, which may be regarded as a circle of infinite radius. Let A and B in Fig. 2 be the pitch-circles of a pair of wheels of unequal diameters, and C a circle say half the diameter of B: in that case a pencil or point at D in the circumference of C rolling inside the circle B into the position C′ will draw a straight and radial line D . E, and that will be the form of the tooth in the wheel B *below* pitch-line. If now this same circle C be made to roll outside the circle A from the position F to the position G, a pencil or point in its circumference at H will describe the epicycloidal curve H . J which will be the form of the tooth of the wheel A *above* pitch-line. It will be observed, that in all

TABLE 3.—Of the PROPORTIONS of TEETH, &c., &c., for SPUR-WHEELS.

Pitch in Inches.	Constant for the Diameter.	IRON-AND-IRON-TEETH.					MORTICE-WHEELS.						
		Length of Teeth.			Thickness of Teeth.	Clearance.	Length of Teeth.			Thickness of Teeth.		Metal at end of Mortice.	Depth of Rim.
		Above pitch.	Below pitch.	Total.			Above pitch.	Below pitch.	Total.	Wood.	Iron.		
1	·3183	·344	·469	·813	·45	·100	·250	·375	·625	·60	·4	·500	1¼
1¼	·3979	·430	·570	1·000	·57	·112	·312	·452	·764	·75	·5	·592	1½
1½	·4775	·516	·669	1·185	·689	·123	·375	·528	·903	·90	·6	·681	1¾
1¾	·5570	·602	·767	1·369	·809	·132	·438	·603	1.041	1·05	·7	·768	2
2	·6366	·688	·865	1·553	·930	·141	·500	·677	1·177	1·20	·8	·853	2¼
2¼	·7162	·774	·951	1·725	1·050	·150	·563	·751	1·314	1·35	·9	·938	2½
2½	·7958	·860	1·058	1·918	1·171	·158	·625	·823	1·448	1·50	1·0	1·020	2¾
2¾	·8753	·946	1·153	2·099	1·292	·166	·688	·895	1·583	1·65	1·1	1·102	3
3	·9549	1·032	1·248	2·280	1·414	·173	·750	·966	1·716	1·80	1·2	1·183	3¼
3¼	1·0345	1·118	1·343	2·461	1·535	·180	·813	1·038	1·851	1·95	1·3	1·263	3½
3½	1·1141	1·204	1·438	2·642	1·657	·187	·875	1·109	1·984	2·10	1·4	1·343	3¾
3¾	1·1936	1·290	1·532	2·822	1·778	·194	·938	1·180	2·118	2·25	1·5	1·421	4
4	1·2732	1·375	1·625	3·000	1·900	·200	1·000	1·250	2·250	2·40	1·6	1·500	4¼
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)

NOTE.—See also the scales in Plate 4, which give the dimensions in this Table by direct measurement.

cases, the part of the tooth of any wheel above pitch-line, works only with the part below pitch-line in its fellow, and *vice versâ;* in our case therefore the curve H . J works with the radial line D . E and will do so correctly. The teeth of the pinion A below pitch may be described on the same principles by a circle K, half the diameter of A rolling inside that circle, into the position K′ when a pencil in its circumference at D, will draw the straight and radial line D . L, and the same circle rolling outside the circle B from the position M to the position N describes the curve O . P, which will work correctly with the radial line D . L.

(8.) The teeth of mortice-wheels are commonly formed precisely in the manner we have described; the teeth have in that case to be shaped and trimmed by hand, and the parts below pitch being flat, can be worked more easily than if they were formed with a hollow curve. The line drawn by a circle rolling inside another of double its own diameter, being always a straight and radial line, of course the trouble of drawing that line by rolling circles may be avoided by using a straight-edge, &c.

(9.) Where the case is not governed by any question of practical convenience, the size of the describing circle is arbitrary. For instance, the circle K rolling from Q to R would describe the hollow curve S . T, which would work with the curve W . X, described by the same circle rolling from U to V. In that case, the whole of the working parts of the teeth in both wheels are drawn by one and the same circle K, applied at K . M . Q . U, and that circle might have been of any other size at pleasure, the form of the teeth varying with every change in the diameter, but still working together correctly.

(10.) The practical millwright can thus obtain almost any desired form of tooth to suit his judgment, by varying the sizes of the describing circles, so long as the great general principle is adhered to—that the same describing circle is used for those parts of the teeth which work together—namely, above pitch in one wheel, and below pitch in the other. It is, however, an almost imperative practical condition, that the describing circle

shall not be larger than half the diameter of the smallest wheel, otherwise the teeth of that wheel would be very small at the root, and consequently very weak.

(11.) When three or more wheels, all of different sizes work together, the case becomes more complicated, but must be treated on the same principles. Thus the wheel I would have its tooth above pitch, formed by the same circle C, by which the tooth of B, below pitch was formed, as at Y . Y′, and the describing circle K, which formed the curve O . P above pitch in the wheel B, will form the tooth below pitch in the wheel I, by rolling inside the pitch-circle from Z to Z′. If a constant or standard describing circle, such as K, be used, the whole matter is simplified.

(12.) The advantages of using a *constant size* of describing circle are very great: for instance, any wheel having its teeth thus formed, will work correctly with any other similar wheel, which is not the case where a variable size of describing circle is used; and again, the trouble of making templets for every different size of rolling circle, and the probability of error by the adoption of a wrong one, is avoided. But to carry this out, the "Standard" size of describing circle adopted must be very small in order to avoid making the teeth of small pinions very weak at the root, and the effect of a very small circle on medium-sized wheels is to give a form of tooth to which there are practical objections on the important points of strength and wearing, as we shall see when we come to the detail of the process of forming the teeth.

(13.) In applying these principles to practice, say to the wheel A, Fig. 2, a portion of the pitch-circle is drawn to the full size on paper, and a templet *a*, formed of wood, say $\frac{1}{10}$th of an inch thick, and shaped to the same radius, is laid upon it. A segment *b* of the describing circle C, &c., is made in the same way, and a needle, &c., is driven through the edge of it obliquely, as at *f*, Fig. 3, so that its point appearing on the lower side just coincides with the curve at *g*, the position on plan being at H in the templet *b*. The templet *a* being kept in position by the pressure of the left hand, the templet *b* is pressed against it

with the right hand with force sufficient to prevent slipping, and is rolled on *a* into the position *c*, the needle at H making a clean indented mark on the paper from H to J. If the needle is fixed at a proper angle, the mark made will not be a scratch, but a clear indentation. By trial, a radius may be found which will draw a curve sufficiently near the exact epicycloid for practical purposes.

The part of the tooth below pitch-line is drawn in the same way by forming a female templet *d*, upon which a segment *e* of the describing circle K is made to roll into the position *h*, and the needle at D draws the line D.L on the paper, &c., &c.

(14.) The details of the method of drawing the teeth of wheels may be more fully illustrated by Fig. 1, in which we have a rack 4 inches pitch, working into a pinion with 10 teeth, having a diameter, by col. 2 of Table 3, equal to $1 \cdot 2732 \times 10 = 12 \cdot 732$, or, say $12\frac{3}{4}$ inches.

For a particular purpose, explained in (5), no clearance between the teeth is allowed in the figure; this, however, will not affect the *form* of the teeth, which is the matter now in hand. If we determine beforehand that the teeth of the pinion below pitch shall be radial, that form would of course be given by a describing circle *a a*, half the diameter of the pitch-circle of the pinion, or $6\frac{3}{8}$ inches, and by the principles already laid down, this circle rolling from *b* to *c* on the straight pitch-line of the rack will draw the curve *d e* or the form of the tooth of the rack above pitch-line. By trial we may now find a radius which will draw that curve with approximate correctness, and we find it to be about $5\frac{1}{2}$ inches, having its centre at *f*, about $1\frac{3}{16}$ inch below the pitch-line: this, however, will only draw the curve correctly from *e* to *h*, the rest, from *d* to *h*, may be drawn from the centre *k* (at the junction of the radius *h*.*f* with the pitch-line of the rack) with the radius *h k*, which is about $1\frac{1}{8}$ inch. It will often happen, as in this instance, that two radii are necessary to obtain a good approximation to the true epicycloidal curve.

(15.) We will assume for the rack that the form of tooth below pitch-line shall be straight and perpendicular to the

pitch-line, which is the form that would be given by a describing circle of infinite radius, or, in other words, by a straight line, and a point in this same straight line rolling on the pitch-circle of the pinion from *m* to *n n*, will draw the curve D *p*, which is the form of the tooth of the pinion, above pitch-line. Searching for a radius by which this curve may be drawn, we find it to be about $3\frac{3}{8}$ inches, having its centre at *r*, about $\frac{1}{10}$th of an inch above the pitch-circle of the pinion; this, however, will draw only that part of the curve from *p* to *s*, leaving about $\frac{1}{4}$th of an inch from *s* to D to be drawn with a radius of about $1\frac{1}{8}$ inch, having its centre on the pitch-circle of the pinion.

(16.) It will be found that the form of tooth with epicycloidal curves varies very greatly with the sizes of describing circle used, and as they are all equally correct in principle, it may seem to be a matter of indifference what particular size of circle is adopted, and what form of tooth ensues. But there are other and practical considerations which are not contemplated in the rules by which the theoretical form of the teeth is fixed, but which it is necessary to remember as affecting the durability of the wheels and the power they are capable of transmitting.

(17.) "*Form as affecting Wear-and-tear.*"—The point of contact between tooth and tooth is always found in the line of the describing circle; for instance, in Fig. 1, it is at E, which is in the circle *a a;* the teeth will therefore be out of contact at the point *t*, where the describing circle and the point of the rack-teeth intersect. When passing the centre at G, the teeth are in contact at that point, or at the pitch-line; while the side of the tooth moves from G to *v*, the point of contact moves from *v* to *t*, the wear-and-tear is thus distributed over the surface *t v*, or more distinctly from *w* to *x*. The relative distance over which the strain is distributed governs the durability of the wheel, and is itself governed by the size of the describing circle; thus with a larger circle *g*, the point of intersection would have been at *i*, and in that case the wear would have been distributed over the reduced distance *i j*, or about $\frac{1}{3}$rd of the distance *t v*.

In some cases this is reduced to a *point*, and the wear-and-tear becomes excessive: this is the case in Fig. 1, with the teeth of the rack below pitch; for the describing circle being one of infinite radius, it coincides with the straight pitch-line, and the teeth are in contact at A . C . D . F. While the wear on the point of the tooth of the pinion is distributed all over the surface from D to p, it is concentrated on the point D of that part of the rack-tooth with which it works, and although the theoretical action is perfect, the teeth of the rack would suffer and very quickly change their form by excessive grinding wear.

(18.) To show how this may be altered by a change in the size of describing circle: say we take the circle a a, by which the point of the rack-tooth was drawn, and cause it to roll from a' to b' when it draws the hollow epicycloidal curve A . c', which will be the new form of the tooth of the rack below pitch. The same describing circle rolling on the pitch-line of the pinion will draw the curve d' e', giving the form of the tooth of the pinion above pitch. The line of contact, instead of being along the straight pitch-line as before, will now be along the describing circle G . f', contact will cease at f', where the describing circle and the point of the pinion tooth intersect, and the wear is distributed over the surface from f' to h' or from A to k' instead of being concentrated on a mere point as in the former case.

(19.) "*Form as affecting the Power of the Wheel.*"—Other things being equal, the power which a wheel can transmit depends on the number of teeth in gear at one time, and as the arc during which contact lasts and consequently the number of teeth in contact depends on the size of the describing circle, this is another important practical point for consideration in selecting the size of the circle to be used in any particular case. Referring again to Fig. 1, with the circle a . a., the arc of contact is G . v., but with a larger circle g, it is G . j., and if the pinion had been one of fine pitch, the result would have been that a greater number of teeth would have been in gear at the same time. This will also be shown by Fig. 7, in which with the small describing circle a the teeth part contact at the point v, but if the pinion had been of larger diameter, a larger

describing circle might have been used say y, and in that case the teeth would have remained in contact to the point x and the arc $u.x$ being double the arc $u.v$ the wheels would been stronger in about the same proportion. In Fig. 1 the effect of using the small circle $a.a.$ instead of the circle of infinite radius, is to reduce the distance in contact from G. A to G. h', &c. But while a small circle thus reduces the number of teeth in contact and thereby the power of the wheel, another effect is to form a stronger tooth, or one having a greater thickness at the root. The general effect of reducing the size of the describing circle is, 1st, to diminish the arc or number of teeth in gear at once, and thereby diminish the power of the wheel; 2nd, to increase the thickness of the tooth at the base and thereby increase the power of the wheel; and 3rd, to increase the wearing surface and the durability of the wheel. The relative amount of each of these can only be determined by trial in each case.

(20.) "*The form of Teeth to match those of an Old Wheel.*"—It frequently happens in practice, that a new wheel has to be made to work with an old one whose teeth are of unusual or irregular form. In such a case, the teeth of the new wheel may be accurately adapted to the old ones by the application of the same principles already explained. To do this, a few teeth of the old wheel should be taken accurately by a sharp pencil or steel scriber on a clean board held against them, &c., and the pitch-circle may be added by the trammel. A wooden templet as at a, Fig. 2, is laid upon it, and another having an *assumed* radius, as a describing circle is rolled upon it, and it is observed how far a mark or point in its circumference will draw a curve coinciding with the actual form of the old tooth. In all probability the size of the describing circle first assumed will not draw the required curve correctly, but it will easily be seen whether it is too large or too small, and another size must be tried until the coincidence is sufficiently near to satisfy the judgment of the operator. Supposing that this has been done for the part of the tooth *above* pitch-line in the old wheel, the describing circle so found will draw correctly the tooth of the

new wheel *below* pitch-line. Then using a concave or female templet as at *d*, Fig. 2, we may find by trial in the same way, a diameter of describing circle that will draw a curve approximating to the form of the old tooth *below* pitch-line, which may then be applied for drawing the tooth of the new wheel above pitch-line, &c.

(21.) The process of forming the teeth by rolling circles is laborious, but it is followed by our best engineers; of course by giving plenty of clearance, and paring away the points of the teeth, wheels with *any form* of teeth may be made to work *somehow*, but where good work is desired it can only be attained by attention to correct principles and the expenditure of time, care, and labour. Professor Willis has endeavoured to reduce the labour by the use of his "Odontograph" which gives by simple curves an approximation to the epicycloidal form drawn by a small standard describing circle, and having therefore the disadvantages which we have shown to belong thereto.

(22.) A careful draftsman may avoid the trouble of making *wooden* templets altogether by using a paper one in the following manner: say we take the case of the Wheel I., Fig. 2; part of the pitch-circle being drawn on the paper to the full size is divided into equal spaces at 0 . 1 . 2 . 3 . 4, &c., &c., say an inch apart more or less, and the same spaces are set off along the edge of the templet cut out of drawing-paper to the radius of the describing circle Y. Then the templet is applied to the pitch-circle, the divisions on its edge being made carefully to coincide with the divisions on the pitch-line, and at each junction, a mark is made with a sharp pencil opposite the division 0 on the templet. Thus when the two Nos. 1 are in contact a mark is made at 1′, when the two Nos. 2 are in contact the mark is at 2′, &c., &c., &c., and thus we get a series of points through which the curve may be drawn by hand, or from which a radius may be found for drawing the curve approximately with the compasses.

(23.) "*Involute Teeth.*"—The involute is a curve formed by the end of a cord unwinding from the circumference of a circle, and the teeth thus formed have one very valuable property which epicycloidal and other forms of teeth do not possess,

namely that the distance of centres of a pair of wheels may be varied so that the teeth are more or less deeply in gear without affecting the regularity of the motion, or the perfect working of the teeth. With epicycloidal teeth the distance of centres must be strictly preserved, and where this can be done, which is the case in most instances, that form of teeth is the best; but with crushing rolls, &c., &c., the distance of centres must be allowed to vary, and in such cases the peculiar property of the involute teeth renders them superior to any other.

(24.) Let W and P in Fig. 11 be the imaginary pitch-circles for a pair of wheels touching at the point A; having determined the proper length of the cog below pitch, set that distance off from A to B on the *larger* wheel of the pair, and with the radius C.B draw the circle D.B.E., which will be the generating circle from which the involute forming the teeth in that wheel are to be drawn. Draw the line A.H. forming a tangent to the circle D.E at H, prolong it to J, and draw the circle K.L., touching that line at M, and that circle will be the generating circle of the involute forming the teeth of the pinion P. A string unwinding from the circle D.E. will draw the involute curve *f.f*, and similarly unwinding from the circle K.L. will draw the involute R.S forming the tooth of the pinion. It will answer the same purpose if a straight-edge be made to revolve against the same generating circles, a point in its edge will draw the same involute curves. The trouble of making templets of the generating circles may be avoided by making equal and corresponding divisions on the straight-edge and generating circles as explained in (22). The length of the tooth *t.u* should not exceed the distance L.P., and similarly the length *v.x.* must not exceed the distance E.W.; they may, of course, be as much shorter than that as may be thought proper.

(25.) It will be observed, that with involute teeth, the curve is a continuous one from base to point. The line of contact, and the direction of the pressure of the teeth on each other, is in the straight line H.A.J, which in small wheels, at least, is very oblique to the direction of motion: this causes a great

pressure on the axes, tending to separate the wheels, and resulting in considerable extra friction on the bearings, and consequent loss of power. This is the great objection to the involute teeth, and for this reason they should seldom be used, except in cases where the distance of centres is variable, when no other form of teeth will work correctly. It should also be observed that the thickness of the teeth should be regulated, so as to be equal at their *bases*, and will in most cases be unequal at the pitch-line, as in Fig. 11.

(26.) "*Teeth of Bevel-Wheels.*"—The teeth of bevel-wheels must be drawn on the same principles as those of spur-wheels, the only important difference being, that the *projected* diameters, and not the real ones, must be taken. Let A., in Fig. 12, be a bevel-wheel, and B. its pinion, say with 54 and 22 teeth respectively, 2½ inches pitch, 6 inches wide on the face of teeth, &c. It is evident that all bevel-wheels have a maximum and a minimum diameter, for instance in our case they are A. *a* and B. *b* respectively; the reputed diameters are always the maximum ones, and thus we find the diameters by col. 2, of Table 3, to be $\cdot 7958 \times 54 = 40$ inches in the wheel, and $\cdot 7958 \times 22 = 17 \cdot 5$ inches in the pinion. These diameters must be drawn upon the board, as in Fig. 12, and the line F.G. drawn through their intersection at D. The line D.E. is perpendicular to the line F.G., and may be drawn by taking any convenient radius, and striking off right and left from the point D, the arcs *m. n.*, and from them as centres drawing other arcs *o. p.* and *v. s.* Then through their intersections the line C.D.E. may be drawn, intersecting the centre lines of the wheels at C and E. Now the *projected* radii E.D. and C.D. are to be taken in drawing the teeth, the projected pitch-line of the teeth of the wheel being D. *e*, drawn from the centre E, and that of the pinion being D. *f*, drawn from the centre C. The whole may then be dealt with precisely as if they were spur-wheels of the new and projected diameters. All the lines of the teeth, both in length and thickness, radiate from the centre F.

ON THE GENERAL PROPORTIONS OF WHEELS, &c.

(27.) "*Length of Teeth.*"—The length of teeth, and the proportions of wheels, are matters for judgment, experience, and taste. Table 3 gives some of the proportions for the teeth of wheels, Table 4 the proportions of the arms, and Plate 4 gives many particulars in a form more direct and useful for practical men, by scales which may be engraved on the back of the common slide-rule, or they may be collected on a pocket-rule specially devoted to them: they can then be applied direct to the work in hand; their use in practice will be illustrated as we proceed.*

With iron-and-iron-toothed wheels, the length of tooth above pitch may be $P \times \cdot 344 = l$, and below pitch $(P \times \cdot 344) + (\sqrt{P} \times \cdot 125) = L$. This gives for clearance between the point of the tooth of one wheel, and the base of its fellow $= \sqrt{P} \times \cdot 125$, which is equal to $\frac{1}{4}$ inch in a wheel 4 inches pitch, and $\frac{1}{8}$ inch in a wheel of 1 inch pitch, &c.: a wheel 4 inches pitch has thus a tooth 3 inches long, or $1\frac{3}{8}$ above pitch, and $1\frac{5}{8}$ below pitch. See cols. 3, 4, and 5 in Table 3, and the scales in Plate 4.

(28.) The teeth of mortice-wheels are commonly made much shorter than those which work iron with iron; the reason being that the wooden-cog has to be made much thicker than its iron fellow, in order to obtain the requisite strength, and the iron one would be excessively weak with the length ordinarily used for iron-toothed wheels. In mortice-wheels, the length of the wooden-cog and its iron fellow, above pitch, may be $P \times \cdot 25$, the clearance at the end of the tooth may be $\sqrt{P} \times \cdot 125$ as in iron wheels, and the length below pitch $(P \times \cdot 25) + (\sqrt{P} \times \cdot 125) = L$. Thus a mortice-wheel, 4 inches pitch, will have $2\frac{1}{4}$ inches for the total length of tooth, or 1 inch above pitch, and $1\frac{1}{4}$ inch below pitch, &c., as per cols. 8, 9, and 10, in Table 3, and the scales in Plate 4.

(29.) "*Thickness of Teeth.*"—Taking first, the case of iron-toothed wheels working together with rough surfaces, as taken

* These scales for wheels may be obtained of Messrs. Spon, 48, Charing Cross, London, price 4*s.*; in ivory, 5*s.* 6*d.* Free by post.

TABLE 4.—Of the BREADTH of ARM at the POINT, for WHEELS with SIX ARMS, DRIVING ONE PINION.

Pitch in inches.	Width on the Face of the Teeth, in Inches.														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Breadth of Arm at the Point, in Inches.														
1	$1\frac{1}{8}$	$1\frac{5}{8}$	$1\frac{7}{8}$	$2\frac{1}{4}$											
$1\frac{1}{4}$	$1\frac{1}{4}$	$1\frac{3}{4}$	$2\frac{1}{8}$	$2\frac{1}{2}$											
$1\frac{1}{2}$	..	$1\frac{7}{8}$	$2\frac{3}{8}$	$2\frac{3}{4}$	3	$3\frac{3}{8}$									
$1\frac{3}{4}$	..	..	$2\frac{1}{2}$	3	$3\frac{1}{4}$	$3\frac{5}{8}$	$3\frac{7}{8}$	$4\frac{1}{8}$							
2	..	..	$2\frac{3}{4}$	$3\frac{1}{8}$	$3\frac{1}{2}$	$3\frac{7}{8}$	$4\frac{1}{8}$	$4\frac{3}{8}$	$4\frac{3}{4}$	5					
$2\frac{1}{4}$	..	..	$2\frac{7}{8}$	$3\frac{3}{8}$	$3\frac{3}{4}$	4	$4\frac{3}{8}$	$4\frac{3}{4}$	5	$5\frac{1}{4}$	$5\frac{1}{2}$				
$2\frac{1}{2}$	..	..	3	$3\frac{1}{2}$	$3\frac{7}{8}$	$4\frac{1}{4}$	$4\frac{5}{8}$	5	$5\frac{1}{4}$	$5\frac{1}{2}$	$5\frac{3}{4}$	6			
$2\frac{3}{4}$	..	..	$3\frac{1}{4}$	$3\frac{3}{4}$	$4\frac{1}{8}$	$4\frac{1}{2}$	$4\frac{7}{8}$	$5\frac{1}{4}$	$5\frac{1}{2}$	$5\frac{3}{4}$	$6\frac{1}{8}$	$6\frac{3}{8}$			
3	..	..	$3\frac{3}{8}$	$3\frac{7}{8}$	$4\frac{1}{4}$	$4\frac{3}{4}$	5	$5\frac{3}{8}$	$5\frac{3}{4}$	$6\frac{1}{8}$	$6\frac{3}{8}$	$6\frac{5}{8}$	$6\frac{7}{8}$	$7\frac{1}{8}$	
$3\frac{1}{4}$	..	..	..	4	$4\frac{1}{2}$	$4\frac{7}{8}$	$5\frac{1}{4}$	$5\frac{5}{8}$	6	$6\frac{3}{8}$	$6\frac{5}{8}$	$6\frac{7}{8}$	$7\frac{1}{4}$	$7\frac{1}{2}$	$7\frac{3}{4}$
$3\frac{1}{2}$	..	..	..	$4\frac{1}{8}$	$4\frac{5}{8}$	$5\frac{1}{8}$	$5\frac{1}{2}$	$5\frac{7}{8}$	$6\frac{1}{4}$	$6\frac{5}{8}$	$6\frac{7}{8}$	$7\frac{1}{8}$	$7\frac{1}{2}$	$7\frac{3}{4}$	8
$3\frac{3}{4}$	..	..	..	..	$4\frac{3}{4}$	$5\frac{1}{4}$	$5\frac{3}{4}$	$6\frac{1}{8}$	$6\frac{1}{2}$	$6\frac{3}{4}$	$7\frac{1}{8}$	$7\frac{1}{2}$	$7\frac{3}{4}$	8	$8\frac{1}{4}$
4	..	..	..	..	5	$5\frac{1}{2}$	$5\frac{7}{8}$	$6\frac{1}{4}$	$6\frac{5}{8}$	7	$7\frac{3}{8}$	$7\frac{5}{8}$	8	$8\frac{1}{4}$	$8\frac{1}{2}$

from the foundry, we must allow a certain clearance between tooth and tooth for errors of workmanship, and other irregularities. The amount of clearance may be taken at $\frac{\sqrt{P}}{10}$ which gives $\frac{1}{10}$th of an inch with 1 inch pitch, and $\frac{2}{10}$ths of an inch with 4 inches pitch, &c.: hence we have $t = \left(P - \frac{\sqrt{P}}{10}\right) \times \cdot 5$, and a wheel 4 inches pitch will have a tooth $(4 - \cdot 2) \times \cdot 5 = 1 \cdot 9$ inches thick, &c., as in col. 6 of Table 3, &c.

With mortice-wheels clearance is unnecessary, the wooden-cog and its iron fellow being usually pitched and trimmed, or shaped accurately by hand. The thickness generally adapted as the dictum of experience, is $\frac{6}{10}$ths the pitch for the wood-cog, and $\frac{4}{10}$ths the pitch for the iron fellow. See cols. 11 and 12 in Table 3, and the scale in Plate 4.

(30.) "*Width on Face of Teeth.*"—The width of face in a wheel is arbitrary, but as a matter of taste there should be a certain proportion between the pitch and the width. A common proportion is to make the width $2\frac{1}{2}$ times the pitch, but this makes a wheel of small pitch too wide, and one of large pitch too narrow, in the judgment of most practical men. A better proportion will be given by the rule $\frac{P^2 \times 1 \cdot 8}{\sqrt{P}} = W$, in which P is the pitch, and W the width in inches; with 1, 2, 3 and 4 inches pitch, this rule gives $1\frac{7}{8}$, $5\frac{1}{4}$, $9\frac{1}{4}$, and $14\frac{1}{2}$ inches, as the respective widths. The same proportions will apply to mortice-wheels; the mortice itself, or shank of the wooden cog, may be about $\frac{1}{4}$ inch narrower than the face of the cog, thus leaving $\frac{1}{8}$ inch for shoulder at each end. The thickness of metal at the end of the mortice will be given by the rule $\frac{P + \sqrt{P}}{4}$ as per col. 13 in Table 3, and the scale in Plate 4; thus a wheel 2 inches pitch, and $5\frac{1}{4}$ inches wide on the face, would have a mortice 5 inches wide, and the thickness of metal at each end being $\cdot 853$, or say $\frac{7}{8}$ inch, the total width would be $6\frac{3}{4}$ inches.

The thickness or depth of rim T, Fig. 9, in mortice-wheels, is usually made equal to the pitch, but a better proportion is given by the rule $P + \cdot 25 = T$, as in col. 14 of Table 3. For really good work the face of the casting should be rough turned before the teeth are fitted in; the expense of turning is nearly compensated by the saving of time and labour in fitting the cogs.

With iron-toothed wheels the thickness of the rim t, Fig. 7, is the same as the thickness of the tooth; the depth of the rib R. projecting inside the rim is $\frac{2}{3}$rds the pitch as given by the scale in Plate 4.

(31.) "*Breadth of Arms*"—No general rule can be given for the number of arms in a wheel: for wheels say 2 inches pitch, four arms may be used for diameter under 2 feet; six arms up to 8 or 9 feet; and eight arms above those sizes. But this will not apply to wheels of other pitches; this is a question to be decided by judgment and taste rather than by rules.

The breadth of the arm at the point, B Fig. 7, will be given by the rule $\frac{7 \cdot 34 \times P \times W \times \sqrt{N}}{A} = B^2$, in which P is the pitch; W the width; A the number of arms; N the number of pinions which the wheel has to drive (usually only one); and B, the breadth of the arm at the point. Thus the wheel Fig. 7, $3\frac{1}{2}$ inches pitch, 10 inches wide, six arms and one pinion, will require an arm $\sqrt{\left(\frac{7 \cdot 34 \times 3 \cdot 5 \times 10 \times \sqrt{1}}{6}\right)} = 6\frac{5}{8}$ inches broad at the point. Table 4 is calculated by this rule. If this wheel had four pinions to drive we should in that case have had $\sqrt{\left(\frac{7 \cdot 34 \times 3 \cdot 5 \times 10 \times \sqrt{4}}{6}\right)} = 9\frac{1}{4}$ inches for the width of the arm at the point.

When a wheel is required to be in halves for convenience in fixing or carriage, &c., the breadth of half the double arm C, in Fig. 7, would be the same as the common arm of a wheel of half the width of face, or in our case 5 inches wide, which by Table 4 is $4\frac{5}{8}$ in. wide. The table gives the breadth of arm for

wheels with six arms only, other cases must be calculated by the rule.

The breadth of arm increases as it approaches the centre; the amount of taper may be for large wheels about $\frac{1}{2}$-inch per foot in length, or $\frac{1}{4}$th inch on each side of the arm: with small wheels it may be $\frac{3}{4}$ inch per foot or $\frac{3}{8}$ inch on each side.

The thickness of the main rib B of the arm Fig. 8 should be equal to the thickness of the iron tooth, and the thickness of the cross ribs *d*, may be $\frac{1}{4}$th the pitch.

(32.) "*Strength of Metal round the Eye.*"—The thickness of metal in the boss of a wheel round the eye, will be given by the rule $\frac{P \times 7}{9} + (\cdot 125 \times D) = T$, in which $P =$ the pitch in inches, $D =$ the diameter of the wheel in feet, and $T =$ the thickness of metal round the eye in inches. The scale in Plate 4 is obtained by this rule, thus for the wheel Fig. 7, measuring with a rule from $3\frac{1}{2}$ inches pitch on one side of the scale, to 6 feet diameter on the other side, we find the required thickness to be $3\frac{3}{8}$ inches; if this wheel had been 10 feet diameter, the thickness would have been $3\frac{7}{8}$ inches; or with 15 feet diameter, $4\frac{1}{2}$ inches, &c., &c. The proper proportions of the key and the key-boss, for strengthening the key-way, are given in Chapter V.

(33.) As a general illustration of the application of the preceeding rules, we have in Fig. 7 a wheel of 64 teeth, working with one pinion of 20 teeth, $3\frac{1}{2}$ inches pitch, &c. The diameters will be found by the rule in (6), &c.: if we determine that the teeth of the pinion below pitch shall be radial, then *a* . *a* will be the diameter of the describing circle, being half the diameter of the pitch-circle *b*. This circle rolling from *c* to *e*, will draw the form of the tooth of the wheel *f* . *f*. The teeth of the wheel below pitch may be drawn by the circle *g*, whose diameter is arbitrary, and the same circle rolling from *h* to *k*, will draw the curve *m* . *m* which is the form of pinion tooth above pitch-line. If the large wheel had been a mortice one as at Fig. 9, then the teeth below pitch being radial, we must have taken the circle *n* half the diameter of the wheel instead of *g*, and the same circle

must have been used instead of $h.k$ for drawing the teeth of the pinion above pitch.

The arm D illustrates the best mode of bolting together a wheel that has to be made in halves. The main bolts E two in number on each side of the boss, should be placed as near the shaft as possible, and to effect that purpose, the arm must have deep bosses as shown: the bolts at F having less strain on them may be of smaller diameter, and should be as near the rim as possible. The wheel has in this case been divided in casting by the plate G, but a better job is secured by planing the two halves and fitting them together thoroughly.

(34.) "*Weight of Wheels.*"—The weight of wheels can be accurately determined only by the number of cubic inches of metal they contain, which multiplied by ·2556 gives the weight in pounds. But this is very laborious, and as in many cases an approximation to the true weight is sufficient for practical purposes, we may give the following rule for finding it:—

TABLE 5.—Of the WEIGHT of WHEEL-CASTINGS.

Diameter at the Pitch-Line.		Pitch.	Width.	Weight.			Value of M.	
ft.	in.	inches.	inches.	cwt.	qrs.	lbs.		
6	7	$3\frac{1}{8}$	$9\frac{1}{4}$	21	2	2	11·7	Iron-toothed Spur-Wheels.
4	$4\frac{1}{2}$	$2\frac{3}{4}$	7	9	3	4	11·7	Ditto.
4	$4\frac{1}{4}$	2	5	5	2	14	12·6	Ditto.
3	$10\frac{3}{8}$	$2\frac{3}{4}$	7	8	2	16	11·66	Ditto.
2	$8\frac{1}{2}$	2	$4\frac{5}{8}$	3	0	10	11·6	Ditto.
2	3	$1\frac{3}{8}$	3	1	1	10	11·6	Ditto.
2	1	$\frac{7}{8}$	$1\frac{1}{2}$	0	1	23	11·54	Ditto.
4	$8\frac{3}{4}$	$3\frac{1}{8}$	9	12	3	2	9·84	Wood-toothed Spur-Wheels.
8	0	$2\frac{3}{4}$	7	18	1	0	12·3	Ditto.
6	4	$3\frac{5}{16}$	$12\frac{3}{4}$	27	1	17	10·8	Iron-toothed Bevel-Wheels.
3	$11\frac{3}{4}$	$3\frac{1}{8}$	$9\frac{1}{4}$	11	0	0	9·74	Ditto.
1	8	$1\frac{1}{2}$	$2\frac{1}{2}$	0	2	24	9·0	Ditto.
1	$2\frac{1}{4}$	$1\frac{3}{8}$	$2\frac{1}{2}$	0	2	3	9·6	Ditto.
7	$0\frac{3}{8}$	$3\frac{5}{16}$	$12\frac{5}{8}$	32	3	3	11·96	Wood-toothed Bevel-Wheels.
4	$5\frac{3}{4}$	$3\frac{1}{8}$	9	11	3	0	9·54	Ditto.

$(D \times P \times W) + (\sqrt{D \times P \times W}) \times M = w$, in which D = diameter at pitch-line in feet, P = pitch in inches, W =

width on face in inches, and w = weight in pounds; thus an iron-toothed spur-wheel 6 feet diameter, 3 inches pitch, and 8 inches wide would weigh about $(6 \times 3 \times 8) + (\sqrt{6 \times 3 \times 8}) \times 12 = 1872$ lbs., or $16\frac{3}{4}$ cwt. The weights of wheels and the corresponding values of M are given by Table 5; the latter may be taken at 12 for iron-toothed spur-wheels, and 13 for spur mortice-wheels. For bevel-wheels 10 may be taken for iron and 11 for wood-toothed wheels. As these are only approximate weights it will be prudent to add a little for contingencies where the maximum weight is desired.

ON THE POWER WHICH WHEELS ARE CAPABLE OF CARRYING.

(35.) The theoretical strength of wheels is a very complicated question; we have to consider, 1st, the strength to sustain a Dynamic load (or a force in motion), from which shocks ariso and back-lash ensues; 2nd, the absolute strength of the teeth to bear a statical load (or dead weight), which is nearly the case with the wheel of a crane, or the rack of a sluice-gate, &c.; and 3rd, the proper proportion of the strain on the teeth to the rubbing surface which bears it, for if a certain pressure per square inch of surface be exceeded, abrasion will ensue, and undue wearing will follow.

(36.) Considering first the ordinary case of wheels in more or less rapid motion, we find that so obscure and complicated are the laws which govern their strength, that it is, perhaps, impossible to give rules based on purely theoretical principles, and we are compelled to refer to experience, and to deduce empirical rules from cases of wheels working well in practice. Mechanical *instinct*, corrected by experience, has guided our practical millwrights (doubtless through a series of failures whose record has been lost) to a sufficient knowledge of the subject to enable them, with more or less success, to fix, by mere judgment, the proper proportions of gearing for the work to be done; it is probable that for every wheel whose sizes are fixed by rule, a thousand are fixed by judgment alone. Perhaps it may appear unnecessary to give rules at all under such circumstances, but it will be found

that there are great differences between the judgments of different engineers, some making their work very much stronger than others, and very much stronger than necessary; and even between the works of the same man, very great differences of strength are to be found, which can only be accounted for by errors or variations of judgment. A rule is therefore necessary, although it be a merely empirical one, and has for its basis the very judgment which it is intended to guide.

(37.) The rule for the power of wheels may take the following form:—$\sqrt{D \times R} \times P^2 \times W \times M = H$, in which

D = Diameter of the wheel at pitch-line in feet.
P = Pitch in inches.
W = Width on the face of teeth in inches.
R = Revolutions per minute.
H = Nominal horse-power.
M = Constant multiplier deduced from cases in practice.

It will easily be seen that when the horse-power is given, and other particulars have to be calculated, the rule becomes—

$$\frac{H}{\sqrt{D \times R} \times P^2 \times W} = M, \text{ and } \frac{H}{\sqrt{D \times R} \times P^2 \times M} = W, \text{ and}$$
$$\frac{H}{\sqrt{D \times R} \times W \times M} = P^2, \text{ and } \left(\frac{H}{\sqrt{D} \times P^2 \times W \times M}\right)^2 = R.$$

The mean value of M may be taken at $\cdot 05$ for wood-toothed, or mortice-wheels, and $\cdot 043$ for iron-and-iron-toothed wheels.

(38.) Thus if we take the case of the wheel shown by Fig. 7, say 6 feet diameter, 25 revolutions, $3\frac{1}{2}$ inches pitch, 10 inches wide, &c.; we have then by our rule $\sqrt{6 \times 25} \times 3\frac{1}{2}^2 \times 10 \times \cdot 043 = 64 \cdot 5$ horse-power; with a mortice-wheel, Fig. 9, the power would have been 75 horse.

These rules will not apply to cases where the velocity is excessively small; in such cases we must ascertain the absolute strength of the teeth as in (46), and it is advisable where the velocity is very small, to calculate the absolute strength of the wheels as a check to their power as calculated by the rules in (37), using of course the rule which brings out the lowest result, or gives the lowest power to the wheel.

(39.) Tables 6, 7, 8, 9 give numerous cases of wheels in practice, and their calculated power by the rules; as might be expected, the coincidence of the actual and calculated power is not perfect, which may arise not only from variations in judgment, but also in many cases from a desire to use existing patterns and save the expense of new ones. For this reason engineers frequently use wheels rather stronger or weaker than they would make them if their judgments were perfectly free. The general agreement of the calculated and actual powers is tolerably satisfactory, thus the combined power of the whole of the wheels in Table 6 (32 in number) is 815 horses, and by calculation 822·3 horses.

(40.) It is remarkable that mortice-wheels should be stronger than iron-toothed ones in carrying power in the ratio of ·05 to ·043, as shown by the rules, in spite of the relative weakness of wood; but in ordinary cases to which these rules apply, it is probable that the strength is regulated by the power of resisting shocks, in which the yielding wood is found superior to the stronger but less elastic iron. It may be, also, that the friction of metal on metal being greater with heavy pressures, and the pressure at which abrasion ensues lower than with wood upon iron, may be the reason of the apparent anomaly. So far as our tables go, they clearly show that practical men have been guided by experience to something like the proportions we have given: the *mean* value of M for iron-toothed wheels in Table 6 is strictly ·0427; for iron-toothed gear to water-wheels in Table 9 it is ·0475, and for mortice-wheels in Table 7 it is ·047. For the latter we have adopted the higher value, ·05, as it agrees better with some of the larger cases in the table, which have been proved by years of successful work to be about correct. All the cases given in the tables have been at work for years satisfactorily, except those we have indicated, and these tend to prove the correctness of the rules which show their comparative weakness. When we come to treat of the *absolute* strength of wheels to resist a dead strain, we shall find that iron-toothed wheels are stronger than wooden-toothed ones for such cases in the ratio of 2·23 to 1; see (48).

TABLE 6.—Of the POWER of SPUR-WHEELS with IRON-TEETH, from CASES in PRACTICE.

Nominal Horse-power.		Revolutions per minute.	Pitch in inches.	Diameter of wheel.		Width on the Face.	REMARKS.
Actual.	Calculated.			ft.	in.	inches.	
60	51·1	19·5	3¼	8	0	9	Paper-mill; wore excessively and broke down.
60	62·4	..	..	..	..	11	Same, made wider, worked better.
50	55·9	40	2¾	15	2⅛	7	Sawing machinery.
46	44·8	17·5	3	12	0	8	Boulton and Watt.
45	41·1	20	3¼	5	0	9	Three-throw deep-well pumps.
44	42·6	25	3¼	4	4⅝	9	Ditto Crystal Palace.
42	44·5	22	3½	5	5	9	Paper-mill.
42	45·4	22	3¼	4	5 13/16	10	Three-throw deep-well pumps.
40	39·2	40	3	3	11¾	8	Paper-mill.
32	30·0	19	3	8	10	6	Boulton and Watt.
30	29·6	24·5	2¾	6	9	7	Asphalte machinery.
30	35·4	22	3¼	2	9	10	Sugar-mill.
*30	27·4	5·76	3½	3	3	12	Ditto.
30	26·7	30	2½	13	1⅝	5	Sawing machinery.
25	22·1	24½	2¾	3	10	7	Paper-mill.
*22	24·3	13·6	3	5	3	7½	Three-throw pumps, Crystal Palace.
20	20·6	36	2½	4	2⅛	6¼	Engineering machinery.
20	18·8	15½	2½	6	10¾	6¾	Three-throw deep-well pumps.
20	23·0	7·28	3	6	8⅛	8½	Ditto.
20	22·0	14·1	2¾	5	9⅛	7½	Ditto.
20	20·3	36	2¼	13	4 7/16	4¼	Sawing machinery.
16	19·0	60	2¼	3	6¼	6	Ditto. New Zealand.
16	15·8	32	2½	3	0⅝	6	Three-throw pumps.
12	12·1	34	2½	1	7⅞	6	Corn mill.
8	10·4	45	2½	1	3⅛	5¼	Ditto.
8	8·8	24	2	3	11⅛	5½	Three-throw pumps.
*7	7·2	30	2	3	3½	4¼	Ditto.
6	6·5	45	1¾	2	1⅝	4½	Ditto.
*5·5	5·5	19	2	3	4⅛	4½	Ditto.
4	4·5	16·4	1¾	3	11⅞	4¼	Ditto.
4	4·2	53	1⅝	1	7⅛	4	Ditto.
1	1·1	88	1⅝	0	7	2	American horse-works.
815	822·3						

NOTE.—The wheels in this table, except those marked *, were first-motion wheels to steam-engines.

TABLE 7.—Of the POWER of SPUR-MORTICE-WHEELS, from CASES in PRACTICE.

Nominal Horse-power.		Revolutions per minute.	Pitch in inches.	Diameter of wheel.		Width on the Face.	REMARKS.
Actual.	Calculated.			ft.	in.	inches.	
42	45·4	22	2¾	18	3¾	6	Centrifugal pump.
30	26·0	24·5	2¼	17	2¼	5	Ditto.
30	31·7	45	2¼	13	10⅛	5	Papier-maché works.
25	24·5	25·5	2¼	14	3⅞	5⅛	Centrifugal pump.
22	18·1	26	2	14	10	4¾	Rope machinery.
16	20·0	26	2¼	13	4 7/16	4¼	Engineering machinery.
15	16·3	30	2	12	3⅝	4¼	Agricultural ditto.
15	13·8	25½	2¼	3	3 1/16	6	Three-throw pumps.
12	14·7	11	2¼	6	9	6½	Ditto.
12	17·1	33	2	12	3⅝	4¼	Crape-weaving machinery.
11	10·6	34	2	2	10⅜	5⅜	Three-throw pumps.
10	14·5	15·2	2¼	5	11⅝	6	Ditto.
10	12·7	16	2¼	5	1⅝	5½	Ditto.
8	9·6	16	2	5	1⅛	5⅜	Ditto.
8	7·0	16	2	2	11⅝	5	Ditto.
6	6·3	45	1½	6	1½	3⅜	Envelope machinery.
4	4·6	53	1½	3	0	3¼	Three-throw pumps.
4	4·4	53	1¾	1	3⅝	3½	Ditto.
3	3·6	65	1¾	0	8⅜	3½	Ditto.
1	1·3	100	1⅛	0	8¼	2½	Washing machinery.
274	302·2						

NOTE.—The wheels in this table were all first-motion wheels to steam-engines. In many cases, the particulars given are not those of the wood-toothed wheel itself, but those of its fellow.

(41.) "*Power of Bevel-wheels.*"—The rules already given apply to bevel-wheels as to others, with certain modifications. Bevel-wheels, as we have stated in (26), have a maximum and a minimum diameter, and also a maximum and minimum pitch, and in calculating their power we must not use the *reputed* sizes which are always the maximum ones, but must ascertain and use the *mean* diameter and the *mean* pitch. Thus, say, we take the case of the wheel A, Fig. 12, with 30 revolutions per minute; the maximum diameter is 3 feet 7 inches, but the minimum diameter *a* is 2 feet 8 inches only, the mean is therefore 3 feet 1½ inches, or 3·125 feet. Again, the maximum pitch

TABLE 8.—Of the POWER of MORTICE-BEVEL-WHEELS, from CASES in PRACTICE.

Nominal Horse-power.		Width on the Face in Inches.	The Driving-wheel.				Pitch in Inches.			Max. Diameter of the Driven Wheel.	Kind of Work.
			Revolutions per Minute.	Diameter.							
Actual.	Calculated.			Maximum.	Minimum.	Mean.	Max.	Min.	Mean.		
				ft. ins.	ft. ins.	ft.				ft. ins.	
40	46·2	$6\frac{1}{2}$	50	9 $0\frac{9}{16}$	8 $0\frac{1}{4}$	8·53	$2\frac{3}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	2 11	Centrifugal Pump.
40	36·2	$5\frac{1}{2}$	148	4 $11\frac{3}{4}$	4 $1\frac{3}{4}$	4·56	$2\frac{1}{2}$	2	$2\frac{1}{4}$	2 $5\frac{1}{2}$	Ditto.
30	21·9	5	164	2 $10\frac{3}{8}$	2 $3\frac{1}{8}$	2·55	$2\frac{1}{4}$	$1\frac{7}{8}$	$2\frac{1}{16}$	2 $9\frac{3}{4}$	Ditto.
24	27·6	$5\frac{1}{2}$	50	7 6	6 $7\frac{3}{4}$	7·073	$2\frac{1}{2}$	$2\frac{1}{8}$	$2\frac{5}{16}$	3 3	Ditto.
20	20·2	$4\frac{1}{2}$	294	2 $6\frac{7}{16}$	1 11	2·228	$2\frac{1}{8}$	$1\frac{5}{8}$	$1\frac{7}{8}$	1 $9\frac{5}{8}$	Ditto.
20	27·3	5	40	9 $11\frac{3}{8}$	9 2	9·558	$2\frac{1}{2}$	$2\frac{1}{4}$	$2\frac{3}{8}$	2 $9\frac{7}{16}$	Ditto.
20	24·5	5	140	4 $0\frac{11}{16}$	3 4	3·70	$2\frac{1}{4}$	$1\frac{7}{8}$	$2\frac{1}{16}$	2 $5\frac{3}{8}$	Ditto.
18	19·9	5	50	6 $7\frac{1}{2}$	5 $10\frac{1}{4}$	6·24	$2\frac{1}{4}$	2	$2\frac{1}{8}$	2 $6\frac{1}{8}$	Ditto.
				*			*			*	

NOTE.—The reputed diameter and pitch of bevel-wheels, by which they are commonly known by practical men, are the maximum ones marked *

is $2\frac{1}{2}$ inches, but the minimum is less in the ratio of the two diameters 43 and 32 inches, it is therefore $\frac{2 \cdot 5 \times 32}{43} = 1 \cdot 86$ inches: the mean pitch is therefore $\frac{2 \cdot 5 + 1 \cdot 86}{2} = 2 \cdot 18$ inches. With these reduced dimensions the power of the wheel by the rule in (37) comes out $\sqrt{3 \cdot 125 \times 30 \times 2 \cdot 18^2 \times 6 \times \cdot 043} = 11 \cdot 8$-horse power. Table 8 gives the strength of bevel-mortice-wheels from cases in practice, which will also serve to llustrate further the mode of calculation.

TABLE 9.—Of the POWER of IRON-TOOTHED FIRST-MOTION SPUR-WHEELS, to WATER-WHEELS, from CASES in PRACTICE.

No.	Nominal Horse-power.		Revolutions per Minute.	Pitch in Inches.	Diameter of Wheel.		Width on the Face.	REMARKS.
	Actual.	Calculated.			ft.	in.	inches.	
1	60	44·2	3·64	$3\frac{1}{4}$	18	0	12	W. Fairbairn (internal).
2	30	34·9	3·14	3	24	0	$10\frac{1}{2}$	From Buchanan.
3	24	20·7	5·09	3	10	$0\frac{1}{2}$	$7\frac{1}{2}$	Overshot-wheel.
4	24	16·0	4·37	$2\frac{1}{2}$	10	6	$8\frac{3}{4}$	Too weak, tooth broke twice, &c.
5	18	18·1	6·00	$2\frac{3}{4}$	12	3	$6\frac{1}{2}$	Breast-wheel.
6	16	12·5	4·95	$2\frac{3}{4}$	8	$3\frac{3}{4}$	6	Ditto (internal).
7	15	19·7	3·00	3	16	2	6	From Buchanan.
8	12	14·2	5·50	$2\frac{3}{4}$	9	$9\frac{1}{2}$	6	Overshot-wheel.
	199	180·3						

(42.) *Tables for calculating the Power of Wheels.*—Tables 10 and 11 have been calculated by the rules in (37), and will greatly facilitate the application of those rules to practice. The following examples will illustrate the mode of using them:—Say we require the power of a wheel 5 feet diameter, 50 revolutions, 2 inches pitch, 5 inches wide. Here $P^2 \times W$ is found by Table 10 to be 20, and $D \times R$ being $5 \times 50 = 250$, we look opposite 20 in the 1st col. of Table 11 for the nearest number to 250, and we find 256 under 14-horse power as an iron-toothed wheel, and

16-horse power as a mortice-wheel. Again, to find the proper pitch and width of a spur-mortice-wheel 14 feet diameter, 20 revolutions, and 50 nominal horse-power, we have $D \times R = 14 \times 20 = 280$, the nearest number to which, under 50-horse power, in Table 11, is 278, which is opposite 60 in col. 1; then looking for 60 in Table 10, we find the wheel might be $2\frac{1}{4}$ pitch 12 inches wide, or $2\frac{1}{2}$ pitch $9\frac{1}{2}$ inches wide, or $2\frac{3}{4}$ pitch 8 inches wide, &c., &c.

(43.) "*Exceptional Cases.*"—Some kinds of machinery require extra strong wheels to resist occasional and momentary strains to which they are subjected; see (55), &c. Thus a 4-ft. millstone takes 4-horse power only, but is found by experience to require with 125 revolutions a spur-mortice pinion $17\frac{5}{8}$ inches diameter, 2 in. pitch, $4\frac{3}{4}$ in. wide $= 12 \cdot 9$-horse power, by the rule. Again, a rag-engine takes 6-horse power only, but is found to require with 160 revolutions a spur-mortice pinion 20 inches diameter, $2\frac{1}{2}$ pitch, 5 in. wide $= 25 \cdot 5$-horse power, &c. The main or first-motion wheels, however, need not be equal to the combined power of all the small driven ones; thus 5 pairs of millstones require wheels of 36-horse power, see (56); and 6 rag-engines, wheels of 66-horse power, see (58).

(44.) "*On the absolute Strength of Wheels.*"—There are many cases to which the preceding rules do not apply, cases where the speed or number of revolutions per minute is practically nothing, such for instance as the rack of a common sluice-gate and the main wheel of a crane. In these cases the absolute strength of the teeth is put to the test, and it may be useful to investigate the case generally; we have here to deal with a dead pressure of so many pounds, &c., and may determine the necessary sizes of the teeth by the common rules for calculating the strength of materials.

"*Crane Wheels,*" *&c.*—Taking the case of the main wheel of a 10-ton crane, say 6 feet diameter with a barrel 1 foot 6 inches diameter measured to the centre of the chain; the pressure at the pitch-line of the wheel is therefore

TABLE 10.—For Calculating the Power of Wheels: to be used in connection with Table 11.

Pitch in Inches.	Width of Wheel on the Face, in Inches.															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	Product of the Pitch Squared (or P²) Multiplied by the Width.															
1	1·00	2·00	3·00	4·00												
1¼	1·56	3·12	4·69	6·25	7·81	9·37										
1½	..	4·50	6·75	9·00	11·2	13·5	15·7	18·0								
1¾	..	6·12	9·18	12·2	15·3	18·4	21·4	24·5	27·6	30·6						
2	..	..	12·0	16·0	20·0	24·0	28·0	32·0	36·0	40·0	44·0					
2¼	..	..	15·2	20·2	25·3	30·4	35·4	40·5	45·6	50·6	55·7	60·7				
2½	..	..	..	25·0	31·2	37·5	43·7	50·0	56·2	62·5	68·7	75·0				
2¾	..	..	..	30·2	37·8	45·4	52·9	60·5	68·1	75·6	83·2	90·7	98·3	106		
3	..	..	..	..	45·0	54·0	63·0	72·0	81·0	90·0	99·0	108	117	126		
3¼	..	..	..	..	52·8	63·4	73·9	84·5	95·1	107	116	127	137	148	158	169
3½	..	..	..	..	..	73·5	85·7	98·0	110	122	135	147	159	171	184	196
3¾	..	..	..	..	..	84·4	98·4	112	126	141	155	168	183	197	211	225
4	..	..	..	..	..	..	112	128	144	160	176	192	208	224	240	256

TABLE 11.—FOR CALCULATING

$P^2 \times W$ from Table 10.	Nominal Horse-power of Mortice-Wheels.										
	1	2	3	4	5	6	7	8	10	12	14
	Nominal Horse-power of Iron and Iron Wheels.										
	·86	1·7	2·6	3·4	4·3	5·2	6·0	7·0	8·6	10	12
	Revolutions per Minute, multiplied by Diameter in Feet.										
2	100	400	900	1600	2500	3600	4900	6400			
3	44·4	177	400	711	1111	1600	2177	2844	4444	6400	8710
4	25	100	225	400	625	900	1225	1600	2500	3600	4900
5	16	64	144	256	400	576	784	1024	1600	2304	3136
7	8·2	32	73	131	204	294	400	522	816	1175	1600
10	4·0	16	36	66	100	144	196	256	400	576	784
15	..	7·1	16	28	44	64	87	114	178	256	348
20	..	4·0	9·0	16	25	36	49	64	100	144	196
25	..	2·6	5·8	10	16	23	31	41	64	92	125
30	..	..	4·0	7·1	11	16	22	28	44	64	87
35	..	..	2·9	5·2	8·2	12	16	21	33	47	64
40	..	..	..	4·0	6·3	9·0	12	16	25	36	49
45	..	..	..	3·2	4·9	7·1	9·7	13	20	28	39
50	..	..	..	2·6	4·0	5·8	7·8	10	16	23	31
60	..	..	..	..	2·8	4·0	5·4	7·1	11	16	22
70	..	..	..	..	..	2·9	4·0	5·2	8·2	12	16
80	..	..	..	..	..	2·3	3·1	4·0	6·2	9·0	12
90	..	..	..	..	..	..	2·4	3·2	4·9	7·1	9·7
100	..	..	..	..	..	..	..	2·6	4·0	5·8	7·8
120	..	..	..	..	..	..	..	..	2·8	4·0	5·4
140	..	..	..	..	..	..	..	..	2·0	2·9	4·0
160	..	..	..	..	..	..	..	..	..	2·2	3·1
180	..	..	..	..	..	..	..	..	..	..	2·4
200	..	..	..	..	..	..	..	..	..	..	..
225	..	..	..	..	..	..	..	..	..	..	..
250	..	..	..	..	..	..	..	..	..	..	..

$\frac{22400 \times 1 \cdot 5}{6} = 5600$ lbs. Let us admit that for safety where life is jeopardized, as in a crane, the safe-load should not be more than $\frac{1}{10}$th of the breaking-strain, and that a cast-iron bar, 1 inch square and 1 inch long, fixed at one end and loaded

THE POWER OF WHEELS.

Nominal Horse-power of Mortice-Wheels.											
16	20	25	30	35	40	50	60	70	80	100	125
Nominal Horse-power of Iron and Iron Wheels.											
14	17	22	26	30	34	43	52	60	69	86	108
Revolutions per Minute, multiplied by Diameter in Feet.											
6400											
4096	6400										
2090	3265	5100	7346								
1024	1600	2500	3600	4900	6400						
455	711	1111	1600	2178	2844	4444	6400	8711			
256	400	625	900	1225	1600	2500	3600	4900	6400		
164	256	400	576	784	1024	1600	2304	3136	4096	6400	
114	178	278	400	544	711	1111	1600	2178	2844	4444	6944
84	131	204	294	400	522	816	1175	1600	2090	3265	5101
64	100	156	225	306	400	625	900	1200	1600	2500	3906
51	79	123	177	242	316	494	711	968	1264	1975	3086
41	64	100	144	196	256	400	576	784	1024	1600	2500
28	44	69	100	136	178	278	400	544	711	1111	1736
21	33	51	73	100	131	204	294	400	522	816	1275
16	25	39	56	76	100	156	225	306	400	625	976
13	20	31	44	60	79	123	178	242	316	494	772
10	16	25	36	49	64	100	144	196	256	400	625
7·1	11	17	25	34	44	69	100	136	178	278	434
5·2	8·2	13	18	25	33	51	73	100	131	204	319
4·0	6·2	9·7	14	19	25	39	56	76	100	156	244
3·2	4·9	7·7	11	15	20	31	44	60	79	123	193
2·6	4·0	6·3	9·0	12	16	25	36	49	64	100	156
2·0	3·2	4·9	7·1	9·7	13	20	28	39	51	79	123
..	2·6	4·0	5·8	7·8	10	16	23	31	41	64	100

at the other as a cantilever, breaks with 6000 lbs., which is the medium strength of cast iron. Assuming 2 inches pitch, 6 inches wide, and using a small describing circle (19) for the purpose of obtaining a tooth strong at the root, we have, as in Fig. 4, a tooth 1·1 inch thick, $1\frac{1}{2}$ inches long, loaded at the end

with a weight W, and may determine the breaking weight by the rule $\frac{t^2 \times B \times 6000}{L} = W$, in which t = the thickness, B = the breadth, and L = the length of the tooth, all in inches, and W = the breaking weight in lbs. In our case this becomes $\frac{1 \cdot 1^2 \times 6 \times 6000}{1 \cdot 5} = 29000$ lbs., and allowing that two teeth are in gear at the same time, we have 58000 lbs., the ratio of which to the real strain is $\frac{58000}{5600} = 10 \cdot 3$ to 1, which is sufficient for safety.

(45.) "*Shrouded Wheels and Racks,*" *&c.*—The effect of shrouding is to reduce the acting length of the cantilever and thereby increase the strength of the teeth. We will take the case of a rack for a common waste-water sluice-gate, say all of oak, the gate 5 feet wide and 10 feet high, with the water level coinciding with the upper edge. The weight of a cubic foot of water being 62·3 lbs. and the *mean* head of water 5 feet, or half the height of the gate, we shall have on 50 square feet area, $62 \cdot 3 \times 5 \times 50 = 15575$ lbs. pressure from the water. Now with wet oak on oak the co-efficient of friction would be about ·7 or $\frac{7}{10}$ths of the insistent weight on the gate, or in our case $15575 \times \cdot 7 = 10900$ lbs., which is the strain upon the teeth of the rack. We may admit that for such a case as this the safe strain may be as much as $\frac{1}{5}$th of the breaking weight. We will assume that the rack may be $2\frac{1}{4}$ pitch, 5 inches wide, and that both the rack and its pinion are shrouded up to the pitch-line, as in Figs. 5, 6; the total length of tooth is $1\frac{3}{4}$ inches, and above pitch $\frac{3}{4}$ inch; but, inasmuch as the tooth would most likely break along the curved line *a a*, the *acting* length to be taken is a mean of those lengths, or $1\frac{1}{4}$ inches. We may now proceed with the calculation as in (44) for ordinary teeth, and we have the strength of two teeth, $= \frac{1\frac{1}{16}^2 \times 5 \times 6000 \times 2}{1 \cdot 25} = 54200$ lbs. breaking weight, the ratio of which to the safe weight is $\frac{54200}{10900} = 5$ to 1, as we determined.

(46.) "*Absolute Strength of Ordinary Wheels.*"—These rules may also be applied to ordinary wheels whose velocity is excessively small, and for which as stated in (38) the rules in (37) may not be applicable. Say we take the case of a pinion 3 feet diameter, $3\frac{1}{4}$ in. pitch, 10 inches wide, making only 3 revolutions per minute, and we have by the rule in (37) $\sqrt{3 \times 3} \times 10 \cdot 5 \times 10 \times \cdot 043 = 13 \cdot 5$ nominal horse-power.

The teeth by Table 3 would be $1 \cdot 535$ inches thick at the pitch-line, but using a small describing circle in forming them, we might obtain, say 2 inches as the thickness at the base, and the length of the tooth being $2 \cdot 461$, its breaking weight by the rule in (44) would be $\frac{2^2 \times 10 \times 6000}{2 \cdot 461} = 97500$ lbs., and if we admit that two teeth are in gear at the same time, we have the strain of 195000 lbs. at the pitch-line, or 19500 lbs. safely. The circumference of 3 feet being 9·4 feet, the velocity is $9 \cdot 4 \times 3 = 28 \cdot 2$ feet per minute, and the nominal horse-power being by (2) $33000 \times 1 \cdot 5 = 50000$ foot-pounds, the wheel will carry $\frac{19500 \times 28 \cdot 2}{50000} = 11$ nominal horse-power only, instead of $13 \cdot 5$-horse power, as found by the rules in (37).

But this rule for absolute strength will not apply to high or even to moderate velocities. For instance, with 27 revolutions, this same pinion would appear by this rule to be 99-horse power, and at 75 revolutions, 275!! horse-power; whereas by the rules in (37) the power comes out only 40 and 67 horse-power respectively. At $4 \cdot 5$ revolutions, or a velocity of 42 feet per minute, the two rules coincide, that is to say for $3\frac{1}{4}$ pitch, as in our present case.

(47.) "*Absolute Strength of Shrouded Wheels.*"—Let Figs. 5 and 6 represent the sections of the rim of a wheel, say 2 feet diameter, making 5 revolutions per minute; then the breaking weight of two teeth would be $\frac{1\frac{1}{16}^2 \times 5 \times 6000 \times 2}{1 \cdot 25} = 54200$ lbs., or 5420 lbs. safe strain, and the velocity at pitch-line being

31·4 feet per minute, the nominal horse-power comes out $\frac{5420 \times 31\cdot4}{50000} = 3\cdot4$ horse. The relative absolute strength of shrouded and unshrouded wheels is governed by the *acting* length of the teeth in the two cases, and admitting the proportions given by Fig. 6, or $1\frac{1}{4}$ to $1\frac{3}{4}$, the absolute strength of shrouded wheels is about 40 per cent. greater than that of ordinary ones.

(48.) "*Absolute Strength of Mortice-Wheels.*"—It appears by (37) and (40) that at high, and even at moderate velocities, mortice-wheels are rather stronger than iron-toothed ones, but their *absolute* strength at low velocities is very much less. The transverse strength of hornbeam, the wood commonly used for mortice-wheels is about $\frac{3}{10}$ the strength of cast iron; a cantilever 1 inch square and 1 inch long will, therefore, break with $6000 \times \cdot3 = 1800$ lbs., (see 44.) Say we take the case of a tooth of a wheel 3 inches pitch, 10 inches wide; by Table 3, col. 11, the wood-cog would be 1·8 inches thick, its length 1·716 inches, and its breaking weight by the rule in (44) $\frac{1\cdot8^2 \times 10 \times 1800}{1\cdot716} = 34000$ lbs. But an iron tooth with the same pitch, would have a thickness of 1·414 inches, and a length of 2·28 inches, and its breaking weight by the same rule would be $\frac{1\cdot414^2 \times 10 \times 6000}{2\cdot28} = 52630$ lbs., the ratio being in this case $\frac{52630}{34000} = 1\cdot55$ to 1. We have here assumed that the iron tooth had the same thickness at the root as at the pitch-line, but by using a small describing circle (19) the thickness at the root might easily be increased to say 1·7 inches, and the breaking weight would then become $\frac{1\cdot7^2 \times 10 \times 6000}{2\cdot28} = 76000$ lbs., and the ratio of the strength of iron and mortice-wheels of the same pitch would be $\frac{76000}{34000} = 2\cdot23$ to 1.

CHAPTER III.

ON SHAFTS.

(49.) "*Power of Shafts.*"—The principal strain to which shafts are subjected is that of torsion. The strength of a cylindrical shaft to resist torsion varies as d^3, and is independent of the length, but the torsional stiffness varies as $\frac{d^4}{L}$; and it is necessary to consider both the strength and stiffness in fixing the proper size for a shaft. A shaft may be strong enough to resist the torsional strain upon it without twisting asunder, but it may be so elastic because of its great length as to be wholly unfit to drive any kind of machinery in which steadiness of motion is essential; or, on the contrary, a shaft may be stiff enough to do its work because of its extreme shortness, but its strength may not be sufficient to resist the twisting strain.

(50.) Before proceeding to apply any rules for fixing the sizes of shafts, it is necessary to consider the conditions under which they are commonly found. Let Fig. 18 be an outline diagram of a Steam-Engine, having its piston A, say 1 foot diameter, and 2 feet stroke, with a pressure of 20 lbs. per square inch, and let the speed be 40 revolutions per minute.

Omitting all considerations of loss by friction, &c., let us see what happens in making 1 revolution. The piston moves twice the length of the cylinder, or 4 feet, and the area of 12 inches diameter, being 113 square inches, the power exerted is $113 \times 20 \times 4 = 9040$ foot-pounds, and the strain on the crank-pin at J is about $113 \times 20 = 2260$ lbs. But the weight B will not be the same as the strain on the crank-pin, although the drum C is the same diameter as the crank-path D, because while the piston moves twice the *diameter* of the crank-path, or 4 feet, the weight B ascends once the *circumference* of the same, or $3 \cdot 14 \times 2 = 6 \cdot 28$ feet, and the weight B will be less than the strain on A, in the ratio of 4 to $6 \cdot 28$.

The power exerted by the piston in raising the weight B varies with every change of position of the crank; at F and G it is nothing, and at H J it is greater than B in the ratio given. The fly-wheel E obviates these irregularities, absorbing the extra power at H and J, and giving it out again at F and G; but it is evident that the shaft K has a greater strain upon it than the shaft L in the ratio $\frac{6 \cdot 28}{4} = 1 \cdot 57$ to 1, and must be made stronger to resist it, although the horse-power is the same in both cases. Thus the strain on the piston is $113 \times 20 = 2260$ lbs., and the work done, $2260 \times 4 \times 40 = 361600$ foot-pounds; the weight B is $\frac{2260}{1 \cdot 57} = 1440$ lbs. nearly, and the work done in raising it is $1440 \times 6 \cdot 28 \times 40 = 361600$ lbs. nearly, or the same as the work done by the piston.

(51.) It becomes necessary, therefore, to make a distinction between steam-engine crank-shafts and ordinary ones. The absolute strength of a shaft varies as we have stated in (49) as d^3, and we have the rules $\frac{d^3 \times R}{M} = P$; $\frac{M \times P}{R} = d^3$; and $\frac{d^3 \times R}{P} = M$: in which P = the nominal horse-power; d = the diameter in inches; and M = a constant derived from experience. The value of M for cast-iron crank-shafts of steam-engines has been given by Buchanan at 400, and for large engines say 30 horse-power, this agrees very well with ordinary practice. Wrought iron being stronger than cast iron for a torsional strain, in the ratio of 14 to 9, we have $\frac{400 \times 9}{14} = 260$ as the value of M for wrought-iron crank-shafts to steam-engines.

Ordinary shafts will have a lower value for M in the ratio of 1 to $1 \cdot 57$, as we have seen; and for cast and wrought iron shafts, these become $\frac{400}{1 \cdot 57} = 254$ for cast iron, and $\frac{260}{1 \cdot 57} = 160$ nearly for wrought iron.

(52.) These rules will do very well for large shafts, say over $4\frac{1}{2}$ inches diameter, but with smaller sizes, although the

absolute torsional strength may be sufficient, the elasticity or angle of torsion becomes so great, that other rules, in which stiffness as well as strength is considered, become necessary. We have stated that the stiffness is governed by $\frac{d^4}{L}$; but if we allow that the angle of torsion may be proportional to the length, which may be admitted in most cases, the power will be in the ratio of d^4 simply, and the length may be disregarded (59).

We have, then, the rules for ordinary wrought-iron shafts, $d^4 \times R \times \cdot 00135 = P$ and $\frac{P}{R \times \cdot 00135} = d^4$; in which P = nominal horse-power, d = diameter of the shaft in inches, and R = revolutions per minute; and thus we have calculated the numbers below $4\frac{1}{2}$ inches in Table 12, which gives the diameter, &c., &c., by inspection.

To avoid a multiplicity of figures, we may adapt the same table to other kinds of shafts, for the power carried will be inversely proportional to their respective multipliers; thus, a cast-iron *crank*-shaft will carry only $\frac{160}{400} = \cdot 4$ of the power of an *ordinary* wrought-iron shaft under otherwise similar conditions of speed, &c., and multiplying the upper numbers by $\cdot 4$, we obtain the corresponding horse-power of cast-iron crank-shafts, as per Table 12. The multipliers for ordinary cast-iron shafts, and for wrought-iron crank-shafts, we found to be respectively 254 and 260, or practically the same value, and they are assumed to be equal in the table.

(53.) Calculating for various sizes, it will be found that with $4\frac{5}{8}$ inches diameter both sets of rules give the same result; but below that size the rule $\frac{P}{R \times \cdot 00135} = d^4$ gives much the larger diameter, and should be used for that reason. Above $4\frac{5}{8}$ the rule $\frac{M \times P}{R} = d^3$ gives the largest sizes, and should be used instead of the other. The fact that the strength and the stiffness follow different laws, necessitates the use of two sets of rules.

Above $4\frac{5}{8}$ inches, shafts whose absolute torsional strength is sufficient to carry the power will be stiff enough to do their work properly; but below $4\frac{5}{8}$, in order to obtain the proper stiffness, the diameter must be larger than necessary to yield the required torsional strength. Table 12 has been calculated by the two sets of rules, in accordance with these principles. Thus, with 16-horse power and 60 revolutions, a common wrought-iron shaft must be $3\frac{3}{4}$ diameter, but if of cast iron, the diameter must be about $4\frac{1}{4}$ inches; a steam-engine crank-shaft for the same power and speed must be $4\frac{1}{4}$ inches diameter in wrought iron, or $4\frac{3}{4}$ inches in cast iron, &c. A cast-iron shaft 9 inches diameter and 22 revolutions, would suffice for the crank-shaft of an engine of 40-horse power, or as an ordinary shaft for 64-horse power. The same shaft in wrought iron would have done for the crank-shaft of an engine of 64-horse power, or as a common shaft for 100-horse power, &c., &c.

(54.) "*Exceptional Cases.*"—These rules and table for steam-engine crank-shafts are strictly adapted only to ordinary cases where the pressure on the piston is nearly uniform; but frequently high-pressure steam is used very expansively, and while the horse-power is governed by the *mean* pressure, the shaft has to bear a much higher or maximum pressure at the commencement of the stroke. Thus, say we had an engine of 20 nominal horse-power, 50 revolutions, and 45 lbs. steam, with a wrought-iron shaft:—in ordinary cases such an engine would require a shaft about $4\frac{5}{8}$ diameter, the mean pressure being pretty nearly 45 lbs. But if this engine were required to work expansively and to cut off steam at $\frac{1}{3}$rd, the *mean* pressure would be reduced to 27 lbs., and the piston must be larger, to compensate for that reduced pressure in the ratio of 45 to 27, and the maximum pressure at the commencement being still 45 lbs., we have a strain at that moment equal to $\frac{20 \times 45}{27} = 33$-horse power, the size of shaft suitable to which we find by Table 12 to be $5\frac{1}{2}$ inches diameter.

With a high-pressure, expansive, and condensing engine, the mean pressure of 45 lb. steam cut off say at $\frac{1}{4}$th would be 21 lbs.,

45	50	60	70	80	90	100	120
T-IRON SHAFTS.							
·6	31·8	38·2	44·6	51	57	64	76
18	20	24	28	32	36	40	48
0	7300	8760	10220	11680	13140	14600	17520
0	5300	6360	7420	8480	9540	10600	12720
5	3950	4740	5530	6320	7110	7900	9480
0	3000	3600	4200	4800	5400	6000	7200
4	2310	2780	3240	3700	4170	4630	5560
4	1815	2180	2541	2904	3267	3630	4360
0	1445	1734	2023	2312	2601	2890	3470
5	950	1140	1330	1520	1710	1900	2280
5	650	780	910	1040	1170	1300	1560
5	455	546	637	728	819	1100	1280
7	330	396	460	530	590	660	790
0	245	294	343	392	441	490	588
7	185	222	259	296	333	370	444
1	145	174	203	232	261	290	348
4	115	138	161	184	207	230	276
1	90	108	126	144	162	180	216
8	75	90	105	120	135	149	180
9	64	77	90	102	115	128	154
0	55	66	77	88	100	111	133
3	48	58	67	77	86	96	115
8	42	50	59	67	76	84	101
3	37	44	52	59	67	74	89
9	33	39	46	52	59	65	78
6	29	35	41	46	52	58	70
1	24	28	33	38	42	47	56
7	19	23	27	30	34	38	46
4	16	19	22	25	28	31	37
1·7	13	15·6	18	21	23	26	31
9·9	11	13·2	15·4	18	20	22	26
8·6	9·5	11·4	13·3	15	17	19	23
7·2	8·0	9·6	11·2	13	14	16	19
5·5	6·0	7·2	8·4	9·6	11	12	14
4·2	4·6	5·6	6·5	7·4	8·3	9·3	11

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With a high-pressure, expansive, and condensing engine, the mean pressure of 45 lb. steam cut off say at $\frac{1}{4}$th would be 21 lbs.,

5	50	60	70	80	90	100	120
T-IRON SHAFTS.							
·6	31·8	38·2	44·6	51	57	64	76
8	20	24	28	32	36	40	48
0	7300	8760	10220	11680	13140	14600	17520
0	5300	6360	7420	8480	9540	10600	12720
5	3950	4740	5530	6320	7110	7900	9480
0	3000	3600	4200	4800	5400	6000	7200
4	2310	2780	3240	3700	4170	4630	5560
4	1815	2180	2541	2904	3267	3630	4360
0	1445	1734	2023	2312	2601	2890	3470
5	950	1140	1330	1520	1710	1900	2280
5	650	780	910	1040	1170	1300	1560
5	455	546	637	728	819	1100	1280
7	330	396	460	530	590	660	790
0	245	294	343	392	441	490	588
7	185	222	259	296	333	370	444
1	145	174	203	232	261	290	348
4	115	138	161	184	207	230	276
1	90	108	126	144	162	180	216
8	75	90	105	120	135	149	180
9	64	77	90	102	115	128	154
0	55	66	77	88	100	111	133
3	48	58	67	77	86	96	115
8	42	50	59	67	76	84	101
3	37	44	52	59	67	74	89
9	33	39	46	52	59	65	78
6	29	35	41	46	52	58	70
1	24	28	33	38	42	47	56
7	19	23	27	30	34	38	46
4	16	19	22	25	28	31	37
1·7	13	15·6	18	21	23	26	31
9·9	11	13·2	15·4	18	20	22	26
8·6	9·5	11·4	13·3	15	17	19	23
7·2	8·0	9·6	11·2	13	14	16	19
5·5	6·0	7·2	8·4	9·6	11	12	14
4·2	4·6	5·6	6·5	7·4	8·3	9·3	11

cast-iron spindle 4 inches diameter, which per table is equal to 36-horse power.

There are doubtless many other kinds of machinery in which excessively strong shafts are found by experience to be necessary. Marine-engine screw-propeller shafts on the contrary seem in practice, to be be made somewhat smaller than our ordinary rules would give; see Appendix, page 80, where special rules are given for calculating their strength.

(56.) Where a number of these extra strong shafts are worked by one main general one, it is not necessary in most cases to make the latter equal in power to the whole combined power of the driven shafts, for the following reason:—The extraordinary strain, to meet which, extra strength is given, occurs only occasionally and irregularly, and is not likely to occur at one and the same moment in all the small shafts. For instance, if we had five pairs of 4-feet millstones in a line, worked by bevel-wheels from one long shaft, the nett power required would be $5 \times 4 =$ 20-horse power; but the power of the five spindles, as we have seen, would be $12 \times 5 =$ 60-horse power. The main shaft need not, however, be equal to 60-horse power, for the reason just given; say we admit that two out of the five pairs of stones are taking for the moment each 12-horse power, and the rest the normal amount of 4-horse power each. Then the first length of shaft next the engine, &c., should be equal to $(12 \times 2) + (3 \times 4)$ $=$ 36-horse power; and if the speed be taken at 30 revolutions per minute, the diameter, per table, would be $5\frac{3}{4}$ inches in wrought iron. After passing the first pair of stones, the power would be $(12 \times 2) + (2 \times 4) =$ 32-horse, and the diameter $5\frac{1}{2}$ inches; the next length would be $(12 \times 2) + (1 \times 4) =$ 28-horse power and $5\frac{1}{4}$ inches diameter, the next $12 \times 2 = 24$ horse and 5 inches diameter, and the last 12 horse and $4\frac{1}{8}$ inches diameter.

(57.) Millstones require very uniform motion to do good work, and for that reason it is not advisable to put more than five or six pairs in a line to be driven by one shaft, as in that case the proper stiffness cannot be obtained without using excessively large sizes, much greater indeed than our table would

give. This objection to a long shaft may be overruled in a great measure by the use of an extra fly-wheel at the extreme end of it, and where the speed is slow, and the size of fly-wheel excessive in consequence of that fact, it may be placed on a quick-running second-motion shaft, &c. The objection to a long shaft does not apply so strongly to cases where the stones are driven by straps instead of wheel-gearing; in that case the stones act as fly-wheels to themselves, which the elasticity of the strap-arrangement allows them to do.

(58.) The same reasoning may be applied in fixing the sizes of long main shafts to rag-engines, saw-mills, &c., &c. In the case of rag-engines the excess of power in the shaft is very great, being $36 - 6 = 30$-horse power each. This excess of strength is required when a large and compact mass of fresh rags gets by carelessness or accident under the roll. This, however, seldom happens, and certainly never in more than one rag-engine at one time. We may therefore admit that 30-horse power should be added to the real nett power on the shaft throughout. Rag-engines are usually driven in pairs from one wheel or pair of wheels close together on the main shaft; say we had three pairs, requiring an engine of 36-horse power. Then the first length next the engine must be equal to $(6 \times 6) + 30 = 66$-horse power; and if the speed be equal to 25 revolutions per minute, the diameter by Table 12 would be $7\frac{1}{2}$ inches. After passing the first pair, the power is reduced to $(4 \times 6) + 30 = 54$-horse power and 7 inches diameter; and the next and last is $(2 \times 6) + 30 = 42$-horse power and $6\frac{1}{2}$ inches diameter. Table 13 gives the sizes of shafts in practice, with the calculated sizes for comparison.

(59.) If it be not admitted, as we assumed in (52), that the angle of torsion may be allowed to increase with the length, but that, on the contrary, all shafts, whether long or short, should be equally stiff, then the diameters necessary become very great. The rule would in that case take the form $\frac{P \times L \times M}{R} = d^4$, and taking the value of M approximately at 15, the diameter necessary for, say, 30-horse power and 150 revolutions for the

TABLE 13.—OF COMMON SHAFTS IN PRACTICE.

Nominal Horse-power.	Revolutions per Minute.	Actual Diam.	Diam. by Table.	KIND OF WORK DONE.	Nominal Horse-power.	Revolutions per Minute.	Actual Diam.	Diam. by Table.	KIND OF WORK DONE.
		inches.					inches.		
1	240	$1\frac{1}{2}$	$1\frac{3}{8}$*	Centrifugal pump (55 ft. long).	15	13·7	7 cast	$6\frac{1}{8}$	Three-throw pumps.
1	88	$1\frac{11}{16}$	$1\frac{11}{16}$	Agricultural machinery.	16	100	$3\frac{1}{4}$	$3\frac{1}{4}$	Paper-mill, &c.
2·7	57	$2\frac{1}{8}$	$2\frac{3}{8}$	Three-throw pumps (too light).	20	284	$3\frac{3}{8}$	3	Centrifugal pump.
4	88	$2\frac{1}{2}$	$2\frac{3}{8}$	Ditto.	20	127	$3\frac{1}{2}$	$3\frac{1}{4}$	General machinery.
4	16	$3\frac{7}{8}$	$3\frac{3}{4}$	Ditto.	20	414	$2\frac{3}{4}$	$2\frac{3}{8}$	Centrifugal pump.
4	20	$4\frac{1}{4}$ cast	4	Ditto.	22	13·6	8 cast	$7\frac{1}{2}$	Three-throw pumps.
4	106	$2\frac{1}{16}$	$2\frac{1}{4}$	Envelope machinery.	24	180	$3\frac{5}{8}$	$3\frac{1}{8}$	Centrifugal pump.
6	104	$2\frac{1}{2}$	$2\frac{1}{2}$	Three-throw pumps.	24	40	$4\frac{3}{4}$	$4\frac{3}{4}$	Paper-making machine.
6	90	$2\frac{5}{8}$	$2\frac{5}{8}$	Agricultural machinery.	25	90	4	$3\frac{3}{4}$	Centrifugal pump.
6	65	$2\frac{7}{8}$	$2\frac{7}{8}$	Envelope machinery.	30	156	4	$3\frac{1}{2}$	Ditto. (144 ft. long).
6	240	$2\frac{9}{16}$	$2\frac{1}{8}$	Centrifugal pump.	30	150	$3\frac{3}{4}$	$3\frac{1}{2}$	General machinery.
7	330	2	2	Ditto.	30	141	$3\frac{5}{8}$	$3\frac{1}{2}$	Sawing machinery.
7	217	$2\frac{1}{4}$	$2\frac{1}{4}$	Ditto.	30	125	4	$3\frac{5}{8}$	Centrifugal pump.
7	110	3	$2\frac{5}{8}$	Three-throw pump.	30	110	3	$3\frac{3}{4}$	Corn-mill, by straps (failed).
7	130	$2\frac{3}{4}$	$2\frac{1}{2}$	Agricultural machinery.	30	110	$3\frac{1}{2}$	$3\frac{3}{4}$	Ditto. (drove pretty well).
8	105	$2\frac{7}{8}$	$2\frac{3}{4}$	Ditto.	40	148	$4\frac{1}{4}$	$3\frac{3}{4}$	Centrifugal pump.
8	110	$2\frac{7}{8}$	$2\frac{3}{4}$	Three-throw pumps.	40	22	8 cast	$9\frac{1}{4}$	Rag-engines, see (58).
8	150	$2\frac{3}{4}$	$2\frac{1}{2}$	Agricultural machinery.	40	152	$4\frac{1}{4}$	$3\frac{3}{4}$	Centrifugal pump.
10	80	3	$3\frac{1}{8}$	Ditto.	40	300	$3\frac{5}{8}$	$3\frac{1}{8}$	Ditto.
10	40	$3\frac{1}{2}$	$3\frac{3}{4}$	Paper-glazing rolls.	40	130	$4\frac{1}{2}$	$3\frac{7}{8}$	Ditto.
10	105	$3\frac{1}{8}$	3	Edge-runners.	42	117	$4\frac{3}{4}$	$4\frac{1}{8}$	Ditto.
12	61	$3\frac{7}{8}$	$3\frac{1}{2}$	General machinery.	42	20	9 cast	$9\frac{3}{4}$	Rag-engines, see (58).
12	150	$2\frac{5}{8}$	$2\frac{3}{4}$	Chaff-cutters, &c.	44	13·6	$8\frac{1}{2}$	8	Three-throw pumps.
14	64	$3\frac{7}{8}$	$3\frac{1}{2}$	General machinery.	50	150	4	4	Sawing machinery.
15	124	$3\frac{3}{8}$	$3\frac{1}{8}$	Agricultural machinery.	60	150	$4\frac{1}{2}$	$4\frac{1}{4}$	Turbine.

* All these shafts are of wrought iron, except otherwise specified; none of them are engine-crank shafts.

lengths 50, 100, 200, 300 feet, come out 3½, 4⅛, 5, 5½ inches respectively. Such a degree of stiffness is in most cases unnecessary, and if it be deemed indispensable, it can be obtained more economically by using a shaft of moderate size, say 3½ inches diameter by Table 12, and fixing a fly-wheel at the extreme end of the shaft itself, or on a quick running counter-shaft, as in (57).

The weight of round and square shafts of wrought iron is given by the following Table.

TABLE of the WEIGHT of ROUND and SQUARE SHAFTS of WROUGHT IRON, 1 foot long.

Size in Inches.	Weight in lbs.		Size in Inches.	Weight in lbs.		Size in Inches.	Weight in lbs.	
	Round.	Square.		Round.	Square.		Round.	Square.
⅛	·042	·053	2⅝	18·2	23·2	7¼	139	177
¼	·166	·211	2¾	20·0	25·5	7½	149	190
⅜	·372	·474	2⅞	21·9	27·9	7¾	159	203
½	·662	·843	3	23·8	30·3	8	169	216
⅝	1·03	1·32	3¼	28·0	35·6	8¼	180	229
¾	1·49	1·90	3½	32·4	41·3	8½	191	244
⅞	2·03	2·58	3¾	37·2	47·4	8¾	203	258
1	2·65	3·37	4	42·4	54·0	9	214	273
1⅛	3·35	4·27	4¼	47·8	60·9	9¼	227	288
1¼	4·14	5·27	4½	53·6	68·2	9½	239	304
1⅜	5·00	6·37	4¾	59·7	76·0	9¾	252	320
1½	5·97	7·58	5	66·2	84·3	10	265	337
1⅝	7·00	8·90	5¼	72·9	92·9	10½	292	372
1¾	8·11	10·3	5½	80·1	102·	11	320	408
1⅞	9·31	11·8	5¾	87·5	111·	11½	350	448
2	10·6	13·5	6	95·3	121·	12	381	486
2⅛	11·9	15·2	6¼	103·	132·	12½	414	527
2¼	13·4	17·1	6½	112·	142·	13	447	570
2⅜	14·9	19·0	6¾	121·	154·	13½	483	614
2½	16·5	21·1	7	130·	165·	14	519	661

ON COUPLINGS FOR SHAFTS.

(60.) "*Couplings.*"—Shafts are not often made more than 20 feet long, from the difficulty and inconvenience in making and fixing them. There are three principal forms of coupling commonly used for round shafts—the solid coupling, with lap-joint; the flange; and the claw coupling.

"*Solid Couplings.*"—The solid coupling shown by Fig. 28 is perhaps the best of all for small shafts up to, say, 4½ or 5 inches diameter; for large shafts they become clumsy and heavy. Table 14 gives the general proportions of these couplings with

TABLE 14.—Of the PROPORTIONS of SOLID HALF-LAP COUPLINGS.

Diameter of Shaft.	Thickness of Metal.	Diameter of Coupling.	Length of Coupling.	Length of Lap.
inches.	inches.	inches.	inches.	inches.
1	1	3	5¼	1
1½	1½	4½	6¾	1⅜
2	1¾	5½	8¼	1¾
2½	2	6½	9¾	2⅛
3	2⅛	7¼	10⅞	2½
3½	2⅜	8¼	12⅜	2⅞
4	2½	9	13½	3¼
4½	2⅝	9¾	14⅝	3⅝
5	2¾	10½	15¾	4

cast-iron boxes, and is calculated by the following rules:— $\sqrt{d} \times 1{\cdot}25 = t$; $D \times 1{\cdot}5 = L$; and $(d \times {\cdot}75) + {\cdot}25 = l$; in which

d = diameter of shaft in inches.
D = „ of coupling „
t = thickness of metal „
L = length of coupling „
l = length of lap „

This kind of coupling requires thoroughly good workmanship, especially in the fitting of the lap-joint; the coupling-box is secured in position by a hollow key, like Fig. 19. The angle or bevel of the joint should be about 1 inch per foot, and may be conveniently described by striking the arcs *c d* and *e f* from the point A with a radius of 6 inches, and setting off ½ inch above and below the centre line, &c., as in the figure.

One great advantage in these couplings over most others, is their safety; there are no projecting bolts, &c., as in the flange coupling, to catch the dress of workmen or work-women, or to become entangled with a strap which comes off accidentally; another is, that when turned and polished, they

are easily kept clean; and there are no bolts to shake loose in working, &c., &c.

(61.) "*Flange Couplings.*"—The flange coupling shown by Fig. 29 is a common and useful kind for small and medium shafts, up to about 6 inches diameter. Table 15 gives the

TABLE 15.—Of the PROPORTION of FLANGE COUPLINGS.

Diameter of Shaft.	Diameter of Flange.	Thickness of Flange.	Diameter of Boss.	Depth at Boss.	No. of Bolts.	Diameter of Bolt.	Diameter of Circle of Bolts.
inches.	inches.	inches.	inches.	inches.		inch.	inches.
1	5	$\frac{3}{4}$	$2\frac{1}{4}$	2	3	$\frac{1}{2}$	$3\frac{1}{2}$
$1\frac{1}{2}$	$6\frac{1}{2}$	$\frac{7}{8}$	$3\frac{1}{4}$	$2\frac{1}{2}$	3	$\frac{5}{8}$	$4\frac{3}{4}$
2	8	$1\frac{1}{16}$	$4\frac{1}{4}$	3	4	$\frac{3}{4}$	6
$2\frac{1}{2}$	$9\frac{1}{2}$	$1\frac{3}{16}$	$5\frac{1}{4}$	$3\frac{1}{2}$	4	$\frac{7}{8}$	$7\frac{1}{4}$
3	11	$1\frac{3}{8}$	$6\frac{1}{4}$	4	4	1	$8\frac{1}{2}$
$3\frac{1}{2}$	$12\frac{1}{2}$	$1\frac{1}{2}$	$7\frac{1}{8}$	$4\frac{1}{2}$	4	1	$9\frac{3}{4}$
4	14	$1\frac{5}{8}$	8	5	6	1	11
$4\frac{1}{2}$	$15\frac{1}{2}$	$1\frac{7}{8}$	$8\frac{7}{8}$	$5\frac{1}{2}$	6	1	$12\frac{1}{2}$
5	17	2	$9\frac{3}{4}$	6	6	$1\frac{1}{8}$	$13\frac{1}{2}$
6	20	$2\frac{1}{4}$	$11\frac{1}{2}$	7	6	$1\frac{3}{8}$	16

general proportions; for good work they should be turned all over, and in any case the two internal faces must be turned to fit together properly; the bolt-holes must be drilled out truly to match one another, and the bolts must be turned parallel throughout, and fit well. To keep the two shafts in line with each other one of them should enter the opposite half-coupling, say $\frac{1}{4}$ inch in the smallest sizes, $\frac{3}{8}$ inch in medium ones, and $\frac{1}{2}$ inch in 6-inch shafts, &c. As these couplings depend for their driving power entirely on the key, that part of the work must be well and firmly done; each half should be secured by a sunk key as in Fig. 21, driven from the inside before the coupling is put together, and cut off flush, &c.; and the face should be turned true in its place after the coupling has been keyed on the shaft.

(62.) There is one objection to the use of solid and flange couplings, more especially for large shafts, namely, their rigidity, and want of accommodation in case of bad adjustment in fixing, where the perfectly straight line is not maintained, or

in other cases where one of the bearings has accidentally worn down considerably more than the rest. With small shafts this is not very important, because a light shaft will spring, and so adjust itself to such irregularities; a heavy shaft may be too stiff to do so, and for that reason heavy shafts more particularly require a yielding coupling, like the following.

(63.) "*Claw Coupling.*"—The claw coupling shown by Figs. 30, 31, should be used for shafts above 6 inches diameter; indeed, there is no reason why it should not be used for small sizes also. When fitted together by chipping, &c., in the usual way, they are very expensive, and require good workmanship; but the expense of this fitting may be entirely avoided by casting one half *upon the other.* In that case one half is cast in sand, &c., in the usual way, and this casting being imbedded in the sand, with the wooden pattern locked into it, a mould is taken from the pattern in the usual way, and the second half is cast upon the first. A perfect fit is thus obtained without labour, and the metal is *chilled* and wears longer, so that this form of coupling is the cheapest of any. Of course, the first half must be coated with founder's blacking where the molten metal of the second half comes in contact with it, to prevent adherence. The shrinkage will be sufficient to enable the two halves to be separated when necessary, but they should be locked together, and so bored to ensure parallelism. This kind of coupling requires well securing to the shaft by good sunk keys, like Fig. 21.

BEARINGS FOR SHAFTS.

(64.) "*Plummer-Blocks or Pedestals.*" — There are many modern variations in the form and general proportions of plummer-blocks; but altogether we think the old-fashioned form shown by Figs. 34 and 35, and moderate proportions, as given by Table 16, are the best for general purposes. For special purposes these proportions may require modification; for instance, where the pressure is very heavy the length may require to be increased to obtain more area. (See 65.) Some engineers

TABLE 16.—Of the PROPORTIONS of PLUMMER-BLOCKS.

Diameter of Bearing.	Length of Bearing.	Height to Centre.	Length of Sole.	Centres of Hold-down Bolts.	Diameter of Bolts.	Size of Holes for Bolts.
inches.	inches.	inches.	ft. in.	ft. in.	inches.	in. in.
$1\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{1}{4}$	0 9	0 7	$\frac{1}{2}$	$\frac{5}{8} \times 1$
2	3	$2\frac{3}{4}$	0 $10\frac{1}{2}$	0 8	$\frac{5}{8}$	$\frac{3}{4} \times 1\frac{1}{4}$
$2\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{1}{8}$	1 0	0 $9\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4} \times 1\frac{1}{4}$
3	4	$3\frac{1}{2}$	1 $1\frac{1}{2}$	0 $10\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8} \times 1\frac{1}{8}$
$3\frac{1}{2}$	$4\frac{1}{2}$	$3\frac{7}{8}$	1 3	0 $11\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8} \times 1\frac{1}{2}$
4	5	$4\frac{1}{4}$	1 4	1 $0\frac{1}{2}$	$\frac{7}{8}$	$1 \times 1\frac{5}{8}$
$4\frac{1}{2}$	$5\frac{1}{2}$	$4\frac{5}{8}$	1 6	1 $1\frac{3}{4}$	1	$1\frac{1}{8} \times 1\frac{3}{4}$
5	6	$5\frac{1}{8}$	1 $7\frac{1}{2}$	1 3	$1\frac{1}{8}$	$1\frac{1}{4} \times 2$
$5\frac{1}{2}$	$6\frac{1}{2}$	$5\frac{3}{4}$	1 9	1 $4\frac{1}{4}$	$1\frac{1}{4}$	$1\frac{3}{8} \times 2\frac{1}{4}$
6	7	$6\frac{1}{2}$	1 $10\frac{1}{2}$	1 5	$1\frac{1}{4}$	$1\frac{3}{8} \times 2\frac{1}{4}$
7	8	$7\frac{1}{2}$	2 1	1 7	$1\frac{1}{2}$	$1\frac{3}{4} \times 2\frac{1}{2}$
8	9	$8\frac{1}{2}$	2 5	1 11	$1\frac{3}{4}$	$2 \times 2\frac{3}{4}$
9	10	$9\frac{1}{4}$	2 $6\frac{1}{2}$	2 0	$1\frac{1}{2}$ Two	$1\frac{3}{4} \times 2\frac{1}{2}$
10	11	10	2 8	2 1	$1\frac{5}{8}$,,	$1\frac{7}{8} \times 2\frac{5}{8}$
11	12	11	2 10	2 3	$1\frac{3}{4}$,,	$2 \times 2\frac{3}{4}$
12	13	12	3 0	2 4	2 ,,	$2\frac{1}{4} \times 3$

have adopted very long bearings $1\frac{1}{2}$ times or even twice the diameter; such proportions are excessively heavy and expensive, and in most cases unnecessary. The principal strain on a shaft is usually a torsional one, and the weight comparatively light; an excessively wide bearing for such a case is a useless expense: it is better, therefore, to adopt moderate proportions for general purposes and to make special wide bearings for exceptional cases. In fact, the width of the bearing should not be governed by the diameter, but by the load or weight which the shaft has to carry. In cases where the upward pressure is great the hold-down bolts may require to be much stronger than in Table 16, and they should be provided with double nuts, as at B, or better still, a double-headed key embracing both nuts at once may be used; but where the upward strain is *very* great the form should be altogether different, such as not to depend wholly on bolts for safety and permanence.

(65.) "*Pressure per Square Inch on Bearings.*"—It has been found by experience that when the pressure per square inch exceeds a certain amount, the oil or other lubricant is squeezed

out, the contact becomes that of metal with metal, the bearing heats, and abrasion ensues in spite of even constant lubrication. The direct experiments of Rennie show that with wrought iron, or steel on cast iron, the friction increases rapidly when the pressure exceeds 5½ cwt. per square inch, and with cast iron on brass, 7 cwt. per square inch. But the most trustworthy data may be obtained from cases in practice; this is done in Table 17, which shows that the pressure on bearings is very variable in

TABLE 17.—Of the PRESSURE on BEARINGS in PRACTICE.

Size of Bearing.		Weight on Bearing.		KIND OF WORK DONE.
Diam.	Length.	Total.	Per square inch in pounds.	
inches.	inches.	lbs.		
9½	11	23500	140	Fly-wheel shaft of 60-horse engine.
8¼	10¼	18800	141	Ditto 40 ditto.
6	7½	32520	458	Crank-pin of 80-horse low-pressure engine.
5	5	16260	414	Link bearing ditto ditto ditto.
5⅜	6½	27720	505	Crank-pin 60-horse ditto ditto.
4½	4½	13860	435	Link bearing ditto ditto ditto.
3½	3½	8550	460	Ditto 60-horse double cylinder ditto.
3	3	5182	366	Ditto 40 ditto ditto.
2¾	2¾	3712	312	Ditto 30 ditto ditto.
2½	2½	2993	304	Ditto 22 ditto ditto.
2⅛	2½	6411	768	Crank-pin 10 high-pressure ditto.
2	2⅜	4564	611	Ditto 6 ditto ditto.
3½	4½	14175	572	Ditto 35 ditto ditto.
3	3¼	11000	718	Bow of deep-well pump to steam-engine beam.
8	3½	8930	203	Head of sling to three-throw pumps.
1½	5¾	8930	660	Pin of ditto ditto.

practice. It may be as high as 750 lbs. per square inch, and yet work tolerably well with proper lubrication; but in most cases it should not exceed 500 lbs., and where it can be done consistently with moderate dimensions the lighter the pressure and the better for the durability of the work. The power consumed by friction is not increased by increasing the length of the bearings, and a wide bearing will wear longer and can be more easily kept cool than a short one. With very high veloci-

ties the pressure per square inch should be very light indeed, otherwise it will be impossible to keep the bearing from heating.

(66.) "*Distance between Bearings.*"—The number and position of bearings must be regulated by the position of the wheels or riggers on the shaft. In all cases the bearings should be as near as possible to the coupling, wheels, &c., &c. But sometimes a long shaft may have no gearing upon it for many feet, and the distance between the bearings must be fixed with reference to the stiffness of the shaft itself. We may admit that a 2-inch shaft, unloaded except by its own weight, may have bearings 10 feet apart, and allowing that the deflection may be in all cases proportional to the distance between bearings, we have the rule $\left(D \times 16\right)^2 = L^3$, and $\frac{\sqrt[2]{L^3}}{16} = D$; in which D = the diameter of the shaft in inches, and L = length between bearings in feet. Table 18 is calculated by this rule, but it must be understood as applying only to cases where the shaft has only its own weight to carry.

(67.) It is sometimes impracticable to get a bearing close to a wheel or rigger; in that case the shaft should be swelled, as in Fig. 24, in which the parts A and B are made of the diameter necessary by Table 12 to carry torsionally the horse-power required, and the central part, carrying the wheel C, is made of much larger diameter, as in the Figure, so as to obtain the requisite stiffness. No rule for the central diameter can be given for such cases; it must be left to the judgment of the engineer.

When a wheel, &c., overhangs the end bearing of a shaft, as in Fig. 25, the neck bearing D has not only to bear the torsional strain but also the transverse strain of the wheel, which tends to wrench the end off. In such cases the bearing D should be made of larger diameter than the rest of the shaft E, as in the Figure, and between D and E the shaft may have the tapered form shown. In such cases the bearing E should be much closer than given by Table 18, say about $\frac{1}{3}$rd of that distance; otherwise the shaft might spring between D and E, and the requisite steadiness of the wheel, &c., would not be

obtained. This is specially important in the case of a *bevel* wheel, where a spring in the shaft would affect the position of the teeth in gear with one another.

TABLE 18.—Of the DISTANCE between BEARINGS of UNLOADED SHAFTS.

Diameter of the shaft in inches..	1	1½	2	2½	3	4	5	6	7	8
Distance between bearings in feet	6·3	8·3	10·1	11·7	13·2	16·0	18·6	21·0	23·2	25·4

CRANK SHAFTS FOR PUMPS.

(68.) "*Crank Shafts for 3-throw Pumps.*"—The strength of a 3-throw crank, Figs. 26, 27, must be calculated to resist two distinct strains to which it is subjected; one being the weight on the buckets due to the head of water, plus the weight of the rods, &c., which tends to break the crank transversely as a beam; and the other, the torsional strain, which tends to *twist* it asunder. In most ordinary cases the diameter required for the torsional strain is the greater of the two, but where the distance between the two bearings, A B, is greater than usual, the diameter necessary to resist the transverse strain may be the greater. The only safe course is to calculate the diameter for both strains, and adopt the one which comes out the largest.

The size of the main bearing A may be determined by the rules already given, or by Table 12, the end bearing may be rather smaller, as it has no torsional strain to resist. From the peculiar form of a 3-throw crank, the torsional strain on the sling bearings C . D . E is greater than that on the main bearing A, in the ratio of 1·73 to 1, the diameter must therefore be greater in the ratio $\sqrt[3]{1 \cdot 73}$ or 1·2 to 1, so that a crank whose main bearing A is 5 inches, will have its sling bearing $5 \times 1 \cdot 2 = 6$ inches diameter, &c.

Wrought iron is by far the most trustworthy material for 3-throw cranks, but the expense of forging is very great, large cranks, 6 or 7 inches in diameter, costing 1*s.* per pound from the

TABLE 19.—Of the SIZES of THREE-THROW CRANKS for PUMPS.

Diameter of Bearings.		NOMINAL HORSE-POWER OF CAST-IRON CRANKS.																							
		·1	·15	·2	·3	·4	·5	·7	1	2	3	4	5	6	8	10	12	14	16	20	25	30	35	40	50
		NOMINAL HORSE-POWER OF WROUGHT-IRON CRANKS.																							
Main Bearing.	Sling Bearing.	·15	·22	·31	·47	·62	·78	1·1	1·5	3·1	4·7	6·2	7·8	9·8	12	15	19	22	25	31	39	47	54	62	78
		REVOLUTIONS PER MINUTE.																							
1½	1·8	23	35	46	68																				
1¾	2·1	12	18	25	37	50	62																		
2	2·4	7	11	15	22	29	37	51	73																
2¼	2·7	..	..	9	14	18	23	32	45																
2½	3·0	..	..	..	9	12	15	21	30	60															
3	3·6	..	..	..	..	..	7	10	14	28	43	57	71												
3½	4·2	..	..	..	..	..	..	..	8	15	23	31	38	46	62										
4	4·8	..	..	..	..	..	..	..	..	9	14	18	23	27	36	46	55								
4½	5·4	..	..	..	..	..	..	..	..	..	8	11	14	17	22	28	34	39	45						
5	6·0	..	..	..	..	..	..	..	..	..	6	8	10	12	16	20	24	28	32	41	51				
5½	6·6	..	..	..	..	..	..	..	..	..	..	6	8	9	12	15	18	21	24	31	38	46	54		
6	7·2	..	..	..	..	..	..	..	..	..	..	..	6	7	9	12	14	16	19	23	29	35	41	47	59
6½	7·8	..	..	..	..	..	..	..	..	..	..	..	..	..	7	9	11	13	15	19	23	28	32	37	46
7	8·4	..	..	..	..	..	..	..	..	..	..	..	..	..	..	7	9	10	12	15	19	22	26	30	38
7½	9·0	..	..	..	..	..	..	..	..	..	..	..	..	..	..	6	7	8	10	12	15	18	21	24	30
8	9·6	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	6	7	8	10	12	15	17	20	25
9	10·8	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	5·5	7·0	8·8	10	12	14	18
10	12·0	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	5·0	6·4	7·6	9	10	13

forge, and as it is impossible to forge them very near the required form, the rough weight and the cost come out greatly in excess of that due to the exact sizes. For this reason, cast iron is very generally used for cranks, and when made of the proper sizes there is not much risk of failure with fair work; very small cranks are usually made of wrought iron. Table 19 gives the proper sizes of cast and wrought iron cranks by inspection, but those sizes it should be observed are for torsional strain only. Table 20 gives the particulars of cranks in practice, with the calculated sizes for comparison. Many of the cases given are of large size, and they have been at work successfully for years.

TABLE 20.—Of THREE-THROW CRANKS in PRACTICE.

Horse-power, Work Done.	Gallons per Minute	Head in Feet.*	Diameter of Bearings.				Revolutions per Minute.	Material.
			Actual.		Calculated.			
			Main.	Sling.	Main.	Sling.		
60·6	800	250	9	10	8·6	10·3	15	Wrought iron.
46·4	600	255	9	10	8·0	9·5	15	Ditto.
36·5	400	301	7½	9	7·45	9·0	14·1	Ditto.
27	450	200	6½	7½	6·46	7·76	16	Ditto.
25·7	314	276	7½	9	7·18	8·64	11·14	Ditto.
16·3	250	215	6	7	5·9	7·0	20	Cast iron.
13	250	170	6½	7½	6·3	7·6	13	Ditto.

(69.) In calculating the diameter to resist the transverse strain the proper method will be best illustrated by an example. Say we have a set of deep-well 3-throw pumps, with barrels 9 inches diameter, 2 feet stroke, and 15 revolutions per minute, fixed in a well 200 feet deep, and raising water to a total height of 330 feet. The modulus of the pumps by Table 1 being ·66, we have $\frac{250 \times 10 \times 330}{33000 \times \cdot 66} = 37 \cdot 8$ indicated, or $\frac{37 \cdot 8}{1 \cdot 5} = 25$ nominal horse-power, and by Table 19 we should require with cast iron, a crank 7½ inches diameter at the main bearing and 9 inches

* The head given includes the friction of the long delivery-pipe.

at the sling bearings, as shown by Figs. 26 and 27; but with wrought iron, the diameter would be $6\frac{1}{2}$ inches, and $7 \cdot 8$ inches respectively. These are the diameters for the torsional strain.

(70.) In calculating for the transverse strain, say the pump-rods are $1\frac{1}{4}$ inch diameter and each 200 feet long; their weight will be 2400 lbs., which being about equally distributed along the length of the beam is equal to a centre load of 1200 lbs.

The head of water can be on two buckets only at one time, because one is always descending unloaded; the central one, having an area of $63 \cdot 6$ square inches, and a column of water 1 foot high, giving a pressure of $\cdot 4327$ lb. per square inch, will have upon it a load of $63 \cdot 6 \times \cdot 4327 \times 330 = 8892$ lbs. If the end sling is half-way between the centre of the crank and the main bearing (which is usually nearly the fact), then the load on it being also 8892 lbs. may be reduced to an equivalent central load of $\frac{8892}{2} = 4446$ lbs., so that the combined central load is $8892 + 4446 = 13338$ lbs. To this has to be added the weight of the rods which we found to be 1200 lbs., and also the extra weight of pump-rod joints, and of the crank itself, say 600 lbs., making thus a total of $13338 + 1200 + 600 = 15138$ lbs.

(71.) A round bar of cast iron, 1 inch in diameter and 1 foot long, breaks with 1334 lbs. in the centre; a wrought-iron one breaks or is crippled with 2000 lbs. The distance between the main bearings A and B is in our case 4 feet 6 inches, as per Fig. 26, and allowing that the working load on a crank should not be more than $\frac{1}{10}$th of the breaking weight, we have $\sqrt[3]{\left(\frac{15138 \times 10 \times 4 \cdot 5}{1334}\right)} = 8$ inches diameter in cast iron, whereas for the torsional strain we found the diameter necessary to be 9 inches, and of course the larger size must be adopted. With wrought iron the diameter of the central sling bearing to resist the transverse strain comes out $\sqrt[3]{\left(\frac{15138 \times 10 \times 4 \cdot 5}{2000}\right)} =$ $7\frac{1}{2}$ inches, instead of $7 \cdot 8$ inches as found necessary for the torsional strain, and here again the larger size must be used. It

will be evident from the foregoing illustration that if for any reason the distance between the bearings A and B in Fig. 26 had been much more than 4·5 feet, the diameter necessary to resist the transverse strain would have been greater than that required for the torsional strain; but in most cases, cranks have nearly the proportions we assumed, and the sizes are governed by the latter.

(72.) In Table 19 the power of cranks is given for very small powers; this is necessary to adapt the table to cases where animal power is used. The real power of an ordinary horse, especially when walking in a circle and raising water by pumps (estimating that power in useful work done) is very much below the standard horse-power of 33000 foot-pounds, see (4), and of course the power of a mule or ass is still less. Table 2 gives the power of horses, &c., raising water by deep-well pumps, which varies with the duration of the labour. Thus when working with pumps eight hours per day, an ordinary horse gives ·354 and an ass ·098 horse-power. But the maximum power exerted for a few moments occasionally under the whip, &c., may be much greater than this; in fact, the maximum power of a horse is four or five times the mean power, and if the machinery was of such a character as to resist a rapid increase in velocity, the strength of shafts, wheels, &c., would have to be adapted to that maximum power. Pumps will give way partially under these circumstances, and we may allow the maximum to be double the mean power. Thus the horse will give $\cdot354 \times 2 = \cdot708$ horse-power, and with 21 revolutions would require, by Table 19, a cast-iron crank $2\frac{1}{2}$ inches diameter at the main bearing, and 3 inches at the sling; and an ass would give a maximum of $\cdot098 \times 2 = \cdot196$, say ·2 horse-power, requiring with fifteen revolutions a cast-iron crank 2 inches at the main, and 2·4 inches at the sling bearings, or a wrought-iron crank $1\frac{3}{4}$ and 2·1 inches diameter, &c., &c.

CHAPTER IV.

ON RIGGERS OR PULLEYS.

(73.) "*Driving Power of Riggers or Pulleys.*"—Let A, Fig. 13, be a rigger or pulley fixed so as to be incapable of turning, and T *t* weights suspended by a strap, E, which passes round the pulley, and may be caused to embrace it more or less by a small guide pulley D. Let now the weight T be increased until the friction of the strap is overcome, and it slips on the pulley, the weight T descending.

The ratio between T and *t* varies—

1st. With the co-efficient of friction of the material of the strap E sliding on the material of the pulley A.

2nd. With the *proportion* which the arc of the pulley embraced bears to the whole circumference of the pulley.

(74.) It is independent of the breadth of the strap *so long as* T *and t remain the same*, but inasmuch as T and *t*, or the strain on the strap, may increase with the breadth, this must not be understood to mean that a narrow strap will drive as much as a wide one; for other things remaining the same, the strain, and therefore the driving power, varies directly and simply as the breadth.

The ratio between T and *t* is also independent of the *diameter* of the pulley, other things remaining the same; thus, for instance, a strap which slips on a pulley 1 foot diameter, with a weight of 1 cwt. at one side, and 2 cwt. at the other, would do the same on a pulley 10 feet or any other diameter, the surfaces being similar. This appears contrary to our instinctive notions, but is quite correct, as I have proved by experiment. But this must not be understood to mean that a small pulley will carry as much power as a large one, for obviously, if both are set in motion, making the same number of revolutions per minute, the relative speeds of strap would be proportional to the diameters, and the power would vary in the same ratio.

(75.) The laws by which the proportion of the entire circumference embraced by the strap governs the ratio of the weights T t are very complicated.

Let F = the co-efficient of friction.
„ L = length of circumference embraced, in feet or inches.
„ R = radius of the pulley, in the same terms as L.
„ T = the greater weight in Fig. 13, &c., or the maximum tension.
„ t = the lesser do. do. minimum tension.

Then $T = t \times (2 \cdot 718)^{\frac{F.L}{R}}$ and $t = \dfrac{T}{(2 \cdot 718)^{\frac{F.L}{R}}}$. These formulæ cannot be worked except by logarithms, and they then take the following forms:—

$$\text{Log } T = \text{Log } t + \left(\cdot 4343 \times \frac{F.L}{R}\right) \text{ and Log } t = \text{Log } T - \left(\cdot 4343 \times \frac{F.L}{R}\right).$$

From Morin's experiments the co-efficients of friction, or the values of F, are as follow:

·47 for leather straps in ordinary working order on drums of wood.				
·28	do.	do.	cast-iron.	
·38	do.	soft and moist	do.	do.
·50 cords or ropes of hemp on pulleys or drums of wood.				

It appears from Morin's experiments that with cast-iron riggers the driving power is the same whether they are turned or not, the adhesion of the strap to the polished surface generating as much friction as with a rough surface.

(76.) If we take the case of a strap in ordinary working order on a cast-iron rigger the value of F will be ·28, and calculating for the four cases shown by Figs. 14, 15, 16, 17, in which the circumference is successively $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and wholly embraced, we find by the first formula that while t = 1 in all cases, T becomes successively 1·553, 2·41, 3·77, and 5·81, as in the Figures. Table 21 is calculated in this way, and gives throughout the

TABLE 21.—Of the RATIO of the STRAINS on the STRAPS of DRIVING RIGGERS, &c.

Ratio of the Arc embraced by the Strap to the entire Circumference.		New Straps on Wooden Drums.		Straps in the Ordinary State on				Soft Straps on Riggers of Cast-iron.		Ropes on Wooden Drums, &c.			
				Wooden Drums.		Cast-iron Riggers				Rough.		Polished.	
	t	T	Q	T	Q	T	Q	T	Q	T	Q	T	Q
·2	1	1 87	·87	1·80	·80	1·42	·42	1·61	·61	1·87	87	1·51	·51
·3	1	2·57	1·57	2·43	1·43	1·69	·69	2·05	1·05	2·57	1·57	1·86	·86
·4	1	3·51	2·51	3·26	2·26	2·02	1·02	2·60	1·60	3·51	2·51	2·29	1·29
·5	1	4·81	3·81	4·38	3·38	2·41	1·41	3·30	2·30	4·81	3·81	2·82	1·82
·6	1	6·59	5·59	5·88	4·88	2·87	1·87	4·19	3·19	6·58	5·58	3·47	2·47
·7	1	9·00	8·00	7·90	6·90	3·43	2·43	5·32	4·32	9·01	8·01	4·27	3·27
·8	1	12·34	11·34	10·62	9·62	4·09	3·09	6·75	5·75	12·34	11·34	5·25	4·25
·9	1	16·90	15·90	14·27	13·27	4·87	3·87	8·57	7·57	16·90	15·90	6·46	5·46
1·0	1	23·14	22·14	19·16	18·16	5·81	4·81	10·89	9·89	23·90	22·90	7·95	6·95
1·5	1	..	..	..	..	..	..	..	..	111·31	110·31	22·42	21·42
2·0	1	..	..	..	..	..	..	..	..	535·47	534·47	63·23	62·23
2·5	1	..	..	..	..	..	..	..	..	2575·30	2574·80	178·52	177·52
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)

value of T when $t = 1$, for different kinds of surface of rigger and states of strap.

When a rope is used, and it is wound more than once round the drum, the frictional power is enormous; thus, with a rough wooden drum and a rope 2·5 times round it, with $t = 1$, T is 2575, &c., &c.

(77.) "*Riggers in Motion.*"—We have so far considered the rigger as fixed; we will now apply the foregoing facts to the case of riggers in motion. The mechanical conditions of a driving rigger with half its circumference embraced by the strap are shown by Fig. 36, in which we have, as before, the rigger A and the weights T and t, as in Fig. 15, where we found them to be respectively 1 and 2·41. But in this case, the rigger A being free to turn, the weights T and t being unequal, there would be no equilibrium without an additional weight at Q, and supposing the drum J to be of the same diameter as the pulley A, it is self-evident that the sum of Q and t must be equal to T, therefore $T - t = Q$, or $2 \cdot 41 - 1 \cdot 0 = 1 \cdot 41 = Q$.

The mechanical power transmitted by the strap, supposing Q to be raised by a rope coiled round the drum as a hoist or windlass, is the *difference* between T and t, and Q might be increased indefinitely, if we could increase T and t indefinitely in the normal proportion; there is, however, a limit to which this can be done, namely, the cohesive strength of the strap by which the heaviest weight, T, is carried. Where leather is used we can obtain the requisite cohesive strength by increasing the width of the strap, or by making it a double or treble one, and this width must in all cases be proportional to T, and not to t or to Q.

In Fig. 36, G may represent the engine-shaft, H its crank, and P the power which is equal to Q. It will be observed that the weight C, or pressure on the bearings due to the tension on the two straps, and also the maximum tension T, is much greater than the power P or weight Q.

(78.) If the weight Q had been 1·0, the maximum tension T would evidently have been $\frac{2 \cdot 41}{1 \cdot 41} = 1 \cdot 71$, and the minimum ten-

sion t, $\frac{1 \cdot 0}{1 \cdot 41} = \cdot 71$, and thus we obtain the strain in Fig. 37; this is the most useful form in which the question can be put, as we thus obtain the proportional maximum strain or width of strap for a unit of power at P.

With a wooden drum the friction of the surfaces is greater, and the strains for the weight Q are different, as is shown by Fig. 38. Here for $t = 1$ we find by Table 21 that T is $4 \cdot 38$, and hence $Q = 4 \cdot 38 - 1 = 3 \cdot 38$, as in the Figure. For Q or P $= 1$, we should have $T = \frac{4 \cdot 38}{3 \cdot 38} = 1 \cdot 29$, and $t = \frac{1}{3 \cdot 38} = \cdot 29$, &c., as in Fig. 39, so that with the same power, P, a strap $1 \cdot 29$ inch wide on a wooden drum would do as well as one $1 \cdot 71$ inch wide on a cast-iron one, as in Fig. 37.

(79.) The case of a rigger of cast iron, with more or less than half the circumference embraced by the strap, is shown by Figs. 40–43.

Thus in Fig. 40 the arc embraced being $\frac{2}{10}$, Table 21 shows (col. 7) that $T = 1 \cdot 42$, and t being $1 \cdot 0$, Q will be $1 \cdot 42 - 1 = \cdot 42$, as in the Figure. For $Q = 1$ we have $T = \frac{1 \cdot 42}{\cdot 42} = 3 \cdot 38$, and $t = \frac{1}{\cdot 42} = 2 \cdot 38$, as in Fig. 41.

With a crossed strap, as in Fig. 42, the arc embraced being $\frac{7}{10}$ths of the circumference, we have $T = 3 \cdot 43$ by Table 21, $t = 1$, and $Q = 2 \cdot 43$; and hence with $Q = 1$ we obtain $T = \frac{3 \cdot 43}{2 \cdot 43} = 1 \cdot 41$, and $t = \frac{1}{2 \cdot 43} = \cdot 41$, &c., as in Fig. 43.

Comparing Figs. 37, 39, 41, and 43 together, it will be seen that with the same engine-power the breadth of strap would be in the ratio $1 \cdot 71, 1 \cdot 29, 3 \cdot 38$, and $1 \cdot 41$.*

* Morin makes the proportional width of strap very different to the above, namely, inversely, as Q in Figs. 36, 38, 40, and 42, and Table 21; this will be shown by our figures, &c., to be an error, the breadth of strap must obviously be proportional to the maximum tension T, and not to the weight lifted Q, or the power P.

(80.) "*On the Power of Riggers.*"—In applying these rules to practice, we might find the safe cohesive strength of leather, gutta-percha, or other material by direct experiment, or we might calculate it from a satisfactory case in practice. Thus let Fig. 37 represent an engine of 5-horse power, making 50 revolutions per minute, with equal cast-iron riggers 5 feet diameter and a single leather strap 7 inches wide. Taking the *nominal* horse-power at $33000 \times 1 \cdot 5$, or 50,000 foot-pounds (2), and the circumference of a 5-feet rigger being $15 \cdot 7$ feet, P and therefore Q will be $\frac{50000 \times 5}{15 \cdot 7 \times 50} = 318$ lbs.; from this we find the maximum tension on the tight side of the strap, or T, to be $318 \times 1 \cdot 71 = 544$ lbs., and the tension on the slack strap, or t, is $318 \times \cdot 71 = 226$ lbs. The strain on the two bearings C in Fig. 36 is $544 + 226 = 770$ lbs.; the maximum strain on the strap is $\frac{544}{7} = 78$ lbs. per inch wide, or say 310 lbs. per square inch of section with leather $\frac{1}{4}$ inch thick.

(81.) It will be more convenient, however, for practice to put the formula into another form, and obtain the necessary constant from experience. Let d = diameter of rigger in inches, w = width of strap in inches, R = revolutions per minute, H = nominal horse-power; M = a constant multiplier from practice, then $\frac{d \times w \times R}{H} = M$; $\frac{M \times H}{d \times R} = w$; $\frac{d \times w \times R}{M} = H$; $\frac{M \times H}{w \times R} = d$; and $\frac{M \times H}{d \times w} = R$.

Thus to find the value from the case we have just given, the rule $\frac{d \times w \times R}{H} = M$ becomes $\frac{60 \times 7 \times 50}{5} = 4200$. This applies only to the case in which the circumference is half embraced by the strap; if the strap had been a double one, we should have taken $2\,w$, &c.

We have seen (78) that the breadth of strap is in all cases proportional to $\frac{T}{Q}$; taking the values of these from cols. 5 to 8

in Table 21, we obtain the numbers in Table 22, and from them we can find the proper breadth of strap in any case. Thus, we

TABLE 22.—Of the PROPORTIONAL WIDTH of STRAPS.

Arc embraced by the Strap.	Power.	Cast-Iron Riggers.		Wooden Drums.	
	Q.	T	t	T	t
·2	1·0	3·38	2·38	2·25	1·25
·3	1·0	2·44	1·44	1·70	·70
·4	1·0	1·98	·98	1·44	·44
·5	1·0	1·71	·71	1·29	·29
·6	1·0	1·53	·53	1·20	·20
·7	1·0	1·41	·41	1·14	·14
·8	1 0	1·32	·32	1·10	·10
·9	1·0	1·26	·26	1·07	·07
1·0	1·0	1·21	·21	1·05	·05
(1)	(2)	(3)	(4)	(5)	(6)

found that a 5-horse engine, with a 5-feet rigger making 50 revolutions, required a strap 7 inches wide, when ·5 of the circumference was embraced; then, by col. 3 in Table 22, with ·2, the breadth must be $\frac{7 \times 3 \cdot 38}{1 \cdot 71} = 13 \cdot 8$ inches. With ·2, ·3, ·4, ·5, ·6, and ·7, the breadth comes out 13·8, 10·0, 8·29, 7·0, 6·28, and 5·77 inches respectively.

(82.) Table 23 is an embodiment of these rules, &c.; and by it we can find w, P, R, or d, by inspection, for arcs between ·3 and ·7, which is sufficient for most cases in practice.

1st. To find the breadth, having the horse-power, revolutions, diameter, and arc embraced given, multiply the diameter in inches by the revolutions, and opposite the given horse-power look for the nearest number thereto, above which and opposite the given arc, will be found the breadth of strap required. Thus, for 10-horse power, with a 5-feet rigger, only ·3 embraced and 100 revolutions, we may find the proper width of single strap, thus:—$d \times$ R is in our case $60 \times 100 = 6000$, which is found opposite 10-horse power in col. 1, under 10 inches wide for cast-iron riggers, or 7 inches wide for a wooden drum, those widths

TABLE 23.—Of the Driving Power of

Ratio of the arc embraced to whole circumference.		WIDTH OF STRAP, IN					
·3		·99	1·49	2·00	2·48	3·00	3·48
·4		·84	1·27	1·69	2·11	2·53	2·95
·5		·75	1·14	1·51	1·89	2·27	2·65
·6		·70	1·06	1·41	1·76	2·11	2·46
·7		·67	1·00	1·34	1·67	2·01	2·34
		WIDTH OF STRAP, IN INCHES, FOR CAST-					
·3		1·43	2·14	2·86	3·58	4·29	5·00
·4		1·18	1·78	2·37	2·96	3·55	4·14
·5		1·00	1·50	2·00	2·50	3·00	3·50
·6		·89	1·34	1·79	2·24	2·69	3·14
·7		·82	1·23	1·65	2·06	2·47	2·88
Nominal Horse-Power.		DIAMETER OF RIGGER, IN INCHES, MULTI-					
Single Leather Strap.	Double Leather Strap.						
1	2	4200	2800	2100	1680	1400	1200
2	4	8400	5600	4200	3360	2800	2400
3	6	12600	8400	6300	5040	4200	3600
4	8	16800	11200	8400	6720	5600	4800
5	10	21000	14000	10500	8400	7000	6000
6	12	25200	16800	12600	10080	8400	7200
7	14	29400	19600	14700	11760	9800	8400
8	16	33600	22400	16800	13440	11200	9600
10	20	42000	28000	21000	16800	14000	12000
12	24	..	33600	25200	20160	16800	14400
14	28	..	39200	29400	23520	19600	16800
16	32	..	..	33600	26880	22400	19200
20	40	..	..	42000	33600	28000	24000
25	50	..	..	..	42000	35000	30000
30	60	..	..	..	50400	42000	36000
35	70	..	..	..	58800	49000	42000
40	80	..	..	..	..	56000	48000
50	100	..	..	..	..	70000	60000
60	120	..	..	..	..	84000	72000
70	140	..	..	..	..	98000	84000
80	160	..	..	..	..	..	96000

RIGGERS OR PULLEYS, with LEATHER STRAPS.

INCHES, FOR WOODEN DRUMS.

4·00	4·47	5·00	6·00	7·00	8·00	9·00	10·00	12·00
3·78	3·80	4·22	5·06	5·91	6·75	7·60	8·44	10·13
3·03	3·41	3·79	4·55	5·30	6·06	6·82	7·58	9·09
2·81	3·17	3·54	4·23	4·94	5·64	6·34	7·05	8·46
2·67	3·01	3·44	4·01	4·68	5·35	6·02	6·69	8·03

IRON RIGGERS, TURNED OR UNTURNED.

5·72	6·43	7·15	8 58	10·00	11·44	12·87	14·30	17·16
4·74	5·33	5·92	7·11	8·29	9·48	10·66	11·85	14·22
4·00	4·50	5·00	6·00	7·00	8·00	9·00	10·00	12·00
3·58	4·04	4·86	5·38	6·28	7·17	8·07	8·97	10·76
3·30	3·71	4·12	4·95	5·77	6·60	7·42	8·25	9·90

PLIED BY REVOLUTIONS PER MINUTE.

1050	934	840	700	600	525	467	420	350
2100	1868	1680	1400	1200	1050	934	840	700
3150	2802	2520	2100	1800	1575	1401	1260	1050
4200	3736	3360	2800	2400	2100	1868	1680	1400
5250	4670	4200	3500	3000	2625	2335	2100	1750
6300	5604	5040	4200	3600	3150	2802	2520	2100
7350	6538	5880	4900	4200	3675	3269	2940	2450
8400	7472	6720	5600	4800	4200	3736	3360	2800
10500	9340	8400	7000	6000	5250	4670	4200	3500
12600	11208	10080	8400	7200	6300	5604	5040	4200
14700	13076	11760	9800	8400	7350	6538	5880	4900
16800	14944	13440	11200	9600	8400	7472	6720	5600
21000	18680	16800	14000	12000	10500	9340	8400	7000
26250	23350	21000	17500	15000	13125	11675	10500	8750
31500	28000	25200	21000	18000	15750	14010	12600	10500
36750	32690	29400	24500	21000	18375	16345	14700	12250
42000	37360	33600	28000	24000	21000	18680	16800	14000
52500	46700	42000	35000	30000	26250	23350	21000	17500
63000	56040	50400	42000	36000	31500	28020	25200	21000
73500	65380	58800	49000	42000	36750	32690	29400	24500
84000	74720	67200	56000	48000	42000	37360	33600	28000

being in both cases opposite ·3, the given arc. If the rigger had been half-embraced, the width would have been 7 and 5·3 inches respectively. With a double strap, opposite 10-horse power in col. 1, 6000 will be found under 5 inches wide for cast-iron riggers, and 3·48 inches for wooden drum ·3 embraced, &c.

2nd. To find the power that a strap will carry; opposite the given arc look for the nearest width of strap, and under that find the number nearest to $d \times R$, opposite to which in column 1 or 2 will be found the horse-power. Say we require the power of a 6-inch strap on a pair of equal cast-iron riggers, 4 feet 3 inches in diameter with 110 revolutions. With equal riggers the arc would be ·5, and we should have $51 \times 110 = 5610$, the nearest number to which under 6 inches wide is 5600 opposite 16-horse as a double strap, or 8-horse as a single one.

Again, to find the power of a 4-inch crossed strap on a cast-iron rigger, 3 feet diameter, 130 revolutions, the arc being say ·6. The nearest width opposite the arc ·6 is 4·04 inches, looking under which for $36 \times 130 = 4680$ we find the nearest number to be 4670 opposite 10-horse power with a double strap, or 5-horse with a single one, &c.

(83.) The best way of ascertaining the arc embraced in any particular case is to draw the outlines of the two riggers at the given distance of centres to scale, and measuring by steps with the compasses the length of strap in inches in contact with the rigger; dividing this by the whole circumference will give the arc required: see the examples given in column 6 of Table 24, &c. The arc required is always that on the smallest rigger of the pair.

With a pair of riggers of unequal dimensions and of the same material the strap will slip first on that rigger whose arc embraced by the strap is the smallest; with an open strap this is always the smallest of the two, except where the conditions are altered by a guide pulley; in calculating the power of riggers, therefore, the small rigger should always be taken as a datum. With a crossed strap, whatever may be the relative diameter of the two riggers, the arc embraced will be the same for both, and

of course it is unimportant which is taken as the basis of calculation.

(84.) Table 24 gives the particulars of many pairs of riggers in practice; the cases of failure are particularly instructive; column 11 shows that in all the cases failure might have been expected. Thus No. 1 required a $10\frac{1}{2}$-inch double leather strap, and a 6-inch gutta-percha one failed to do the work. In No. 2 a larger rigger was substituted, a 7-inch double leather strap should have been used, and the 6-inch gutta-percha one did the work badly. No. 4 failed with a 9-inch single strap to do the work for which a 14-inch single or 7-inch double strap was required. No. 6 required a 10-inch single or 5-inch double strap, and failed to do the work with a 6-inch single strap. No. 8 required a 13-inch single or $6\frac{1}{2}$-inch double strap and failed with an 8-inch single strap, &c. It will be observed that in cases Nos. 1 and 11 the difficulty was overcome by using larger riggers; and in cases No. 4, 6, and 8 by converting the single strap into a double one. Circular saws and some other kinds of machinery require extra strength of strap, as shown by No. 18, and explained in (55), &c.

The rules and Table we have given apply strictly to *leather* straps only; leather is every way the best material, and is not likely to be permanently superseded by the new materials, gutta-percha, india-rubber, &c.

Table 24 shows that the power of a gutta-percha strap $\frac{5}{16}$ or $\frac{3}{8}$ inch thick is from 25 to 50 per cent. greater than that of a single leather one. We found in (80) that in practice leather straps bear about 310 lbs. per square inch of section, and we may allow that gutta-percha will bear about 400 lbs. From direct experiments the cohesive strength of gutta-percha is 15 cwt. or 1682 lbs. per square inch, as shown by Table 25, which also gives the extensions by different weights. It will be observed that with weights greater than 3 cwt. per square inch the extensions increase rapidly, showing that the material is overstrained; the Table gives the mean result of two experiments.

TABLE 24.—Of the DRIVING-POWER of RIGGERS

No.	Nominal Horse-power.	Diameter of the Two Riggers. Driver.		Driven.		Distance of Centres.		Arc on Smallest Rigger.	Revolution of Smallest Rigger.	Arrangement of Strap.
		ft.	in.	ft.	in.	ft.	in.			
1	10	5	0	2	6	8	0	·446	80	Open
2	10	7	0	3	6	8	0	·424	80	Ditto
3	10	10	0	2	5	30	0	·472	165	Ditto
4	12	7	0	7	0	32	0	·58	36	Crossed
5	12	7	0	7	0	32	0	·58	36	Ditto
6	6	5	0	5	0	9	3	·5	40	Open
7	6	5	0	5	0	9	3	·5	40	Ditto
8	18	12	6	3	6	20	0	·424	152	Ditto
9	18	12	6	3	6	20	0	·424	152	Ditto
10	18	6	3	1	3	19	6	·457	770	Ditto
11	3	1	3	2	0	12	0	·49	90	Open
12	3	2	0	3	2	12	0	·48	90	Ditto
13	20	9	0	5	0	16	0	·457	65	Ditto
14	12	10	0	3	0	16	6	·43	113	Ditto
15	12	3	0	4	9	11	0	·473	100	Ditto
16	12	10	0	3	0	14	0	·42	133	Ditto
17	10	6	0	4	5	14	6	·6	49	Crossed
18	10	4	3	2	2	11	3	·6	463	Ditto
19	10	8	0	5	0	20	0	·476	64	Open
20	6	7	0	3	6	13	0	·46	106	Ditto
21	6	9	0	3	6	11	0	·416	128	Ditto

PROPORTIONS OF RIGGERS.

(85.) "*The Proportions of Arms, &c., of Riggers.*"—The number, form, and strength of the arms, &c., of riggers is important, not only as a matter of taste, but also as affecting the safety in casting, for if the proper proportion between the strength of the arms and rim be not preserved, one or other is very likely to fly in cooling or to give way when set to work. Curved arms are the safest in this respect, their form permitting them to give

OF PULLEYS, from CASES in PRACTICE.

Particulars of Strap.		Calculated Width of Strap.	Remarks.
Width.	Material.		
inches.			
6	Gutta-percha	10½ double	Failed entirely, riggers made larger. See No. 2.
6	Ditto	7 ditto	Badly, required resin, and gave trouble.
6	Ditto	4½ ditto	Drove well, circular 3 ft. 6 in. saw. Same engine as No. 1.
9	Single leather	14 single	Failed, strap altered to double one. See No. 5.
9	Double ditto	7 double	Drove well.
6	Single ditto	10 single	Failed, strap altered to double one. See No. 7.
6	Double ditto	5 double	Drove well.
8	Single ditto	13 single	Failed, strap altered to double one. See No. 9.
8	Double ditto	6½ double	Drove well.
6½	Gutta-percha $\frac{5}{16}$ thick	7¾ single	Drove, but not well. Same engine as No. 8, &c.
6	Single leather	9½ ditto	Failed, riggers made larger. See No. 12.
6	Ditto	6 ditto	Drove well.
9	Double leather	11 double	
9	Ditto	5½ ditto	Drove well, riggers lagged with wood.
8	Ditto	7½ ditto	Drove well.
6	Ditto	6 ditto	Ditto.
7	Ditto	7½ ditto	Ditto.
4	Ditto	3½ single	Ditto, extra strong, for 4ft. 6in. circular saw.
9	Gutta-percha	5½ double	Drove well.
5¾	Ditto, ¼ thick	6½ single	Ditto.
5½	Single leather	5½ ditto	Ditto.

way and allow the rim to contract in the casting. The particular curves to be used are arbitrary to some extent, depending on taste; a good form is shown by Fig. 32, and may be described as follows:—Draw the line A · B, and the line C . D perpendicular to it; from the point E on the line A · B, with a radius of $\frac{1}{5}$th the diameter of the rigger, draw the curve F, and from the point G in the line C . D, with a radius $\frac{1}{7}$th of the diameter, draw the curve H, and we thus obtain the centre lines of one of the arms.

TABLE 25.—Of the COHESIVE STRENGTH and ELASTICITY of OLD GUTTA-PERCHA STRAPS, &c.

Strain per Square Inch in Cwts.	Length.	Extension per Cwt.	Permanent Sett.	Strain per Square Inch in Cwts.	Length.	Extension per Cwt.	Permanent Sett.
0	1·000	·000		9	1·100	·0111	
1	1·007	·007		10	1·120	·012	
2	1·014	·007		11	1·142	·013	
3	1·021	·007		0	1·029	..	·029
4	1·029	·00725		12	1·182	·0152	
5	1·040	·008		13	1·210	·0161	
6	1·051	·0085		14	1·265	·0190	
7	1·064	·00914		0	1·064	..	·064
0	1·007	..	·007	15 Broke.	1·287	·0192	
8	1·075	·00937					

(86.) Straight arms are generally preferred to curved ones, and when properly proportioned will cause no trouble from contraction in casting. The best form of section of the arm is an oval, but should not be a true ellipse, which has a heavy appearance; it should be drawn with a single curve having a radius about $\frac{5}{8}$ths of the width of the arm, and the thickness may be half the breadth, as in Fig. 33, the sharp points at A and B being rounded off. The number of the arms is arbitrary, but in most cases four will suffice for diameters under 18 inches, six may be used up to 8 or 9 feet, and eight for larger diameters.

(87.) "*Strength of Arms.*"—Admitting that the *thickness* of arm is in all cases half the breadth, as in Fig. 33, its strength will vary as the cube of the breadth, and we have the following rules, $\frac{d \times w}{N \times 4} = B^3$ and $\frac{d \times w}{N \times 8} = b^3$, in which—

d = the diameter of the rigger in inches.
w = width on the rim of ditto, in ditto.
B = breadth of arm at base, in ditto.
b = breadth ditto point, in ditto.
N = number of arms.

Table 26 gives a general comparison of this rule with approved proportions of riggers in practice.

TABLE 26.—Of the PROPORTIONS of RIGGERS in PRACTICE.

Diameter.	Width.	Breadth and Thickness of Arm at Base.		Breadth and Thickness of Arm at Point.		No. of Arms.	Depth of Rim in Centre.	Remarks.
		Actual.	Calculated.	Actual.	Calculated.			
ft. in. 2 0	7	$1\frac{7}{8} \times 1$	$1\frac{7}{8} \times 1$	$1\frac{5}{8} \times \frac{7}{8}$	$1\frac{9}{16} \times \frac{13}{16}$	6	$1\frac{3}{8}$	Approved proportions.
2 8	7	$2\frac{1}{4} \times 1$	$2\frac{1}{8} \times 1\frac{1}{16}$	$1\frac{3}{4} \times 1$	$1\frac{11}{16} \times \frac{7}{8}$	6	$1\frac{3}{8}$	Ditto.
3 10	$9\frac{1}{2}$	$2\frac{1}{2} \times 1\frac{1}{8}$	$2\frac{5}{8} \times 1\frac{1}{4}$	2×1	$2\frac{1}{8} \times 1\frac{1}{16}$	6	$1\frac{1}{2}$	Ditto.
4 0	9	$2\frac{3}{8} \times 1\frac{1}{8}$	$2\frac{5}{8} \times 1\frac{1}{4}$	$1\frac{7}{8} \times 1$	$2\frac{1}{8} \times 1\frac{1}{16}$	6	$1\frac{1}{2}$	Ditto.
4 3	5	$2 \times \frac{7}{8}$	$2\frac{3}{16} \times 1\frac{1}{16}$	$1\frac{5}{8} \times \frac{7}{8}$	$1\frac{3}{4} \times \frac{7}{8}$	6	$1\frac{1}{2}$	Ditto.
4 8	5	$2 \times \frac{15}{16}$	$2\frac{5}{16} \times 1\frac{1}{8}$	$1\frac{1}{2} \times \frac{7}{8}$	$1\frac{7}{8} \times \frac{7}{8}$	6	$1\frac{1}{2}$	Ditto.
4 8	10	$2\frac{3}{4} \times 1\frac{1}{4}$	$2\frac{7}{8} \times 1\frac{3}{8}$	2×1	$2\frac{5}{16} \times 1\frac{1}{8}$	6	$1\frac{5}{8}$	Ditto.
5 0	$4\frac{1}{2}$	$2\frac{1}{4} \times 1\frac{1}{8}$	$2\frac{1}{4} \times 1\frac{1}{8}$	$1\frac{3}{4} \times \frac{7}{8}$	$1\frac{3}{4} \times \frac{7}{8}$	6	$1\frac{3}{8}$	Ditto.
7 0	13	$3\frac{7}{8} \times 1\frac{7}{8}$	$3\frac{9}{16} \times 1\frac{3}{4}$	$3 \times 1\frac{1}{2}$	$2\frac{13}{16} \times 1\frac{3}{8}$	6	$2\frac{3}{4}$	Ditto.
8 0	14	$4\frac{1}{4} \times 2\frac{1}{2}$	$3\frac{9}{16} \times 1\frac{3}{4}$	$3\frac{1}{4} \times 2$	$2\frac{13}{16} \times 1\frac{3}{8}$	8	3	Too strong in the arm.
9 0	9	$3\frac{5}{8} \times 1\frac{7}{8}$	$3\frac{1}{8} \times 1\frac{1}{2}$	$2\frac{5}{8} \times 1\frac{3}{8}$	$2\frac{1}{2} \times 1\frac{1}{4}$	8	$2\frac{1}{4}$	Approved proportions.
9 0	10	$4\frac{1}{4} \times 2\frac{1}{4}$	$3\frac{9}{16} \times 1\frac{3}{4}$	$3\frac{3}{8} \times 1\frac{11}{16}$	$2\frac{13}{16} \times 1\frac{3}{8}$	6	$2\frac{1}{2}$	Too strong in the arm.

(88.) "*Strength of the Metal round the Eye.*"—The strength of metal in the boss varies not only with the diameter of the rigger, but also with the size of the shaft. Putting D for the diameter of the rigger in feet, d for the diameter of the shaft in inches, and t for the thickness of metal in $\frac{1}{8}$ths of an inch, we have the rule $D + d + 5 = t$. Table 27 has been calculated by this rule, which has been found to agree very well with practice. The proper size of key and form of key-boss, &c., for riggers may be found by reference to Chapter V.

TABLE 27.—Of the THICKNESS of METAL ROUND the EYE of RIGGERS.

Diameter of Rigger, in Feet.	DIAMETER OF SHAFT, IN INCHES.					
	1	2	3	4	5	6
	THICKNESS ROUND EYE, IN INCHES.					
1	$\frac{7}{8}$	1	$1\frac{1}{8}$	..	..	..
2	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$
3	..	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{3}{4}$
4	..	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{3}{4}$	$1\frac{7}{8}$
5	..	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{3}{4}$	$1\frac{7}{8}$	2
6	..	..	$1\frac{3}{4}$	$1\frac{7}{8}$	2	$2\frac{1}{8}$
7	..	..	$1\frac{7}{8}$	2	$2\frac{1}{8}$	$2\frac{1}{4}$
8	..	..	2	$2\frac{1}{8}$	$2\frac{1}{4}$	$2\frac{3}{8}$
9	..	..	$2\frac{1}{8}$	$2\frac{1}{4}$	$2\frac{3}{8}$	$2\frac{1}{2}$

(89.) The experiments of Morin show, as we have seen (75), that the power transmitted by cast-iron riggers is the same whether the rim be turned or not; but for rapid velocities it is necessary to turn the rim both inside and outside for the purpose of preserving the balance. When a rigger, &c., is out of balance, which will generally be the case as it leaves the sand, it exerts a violent shaking action on any machine to which it is attached, which may be injurious and even destructive.

(90.) "*Rounding Face for Riggers.*"—Where there is no guide lever the surface should be turned rounding, say to the extent of $\frac{1}{2}$ inch per foot in width of rim; the effect of this is to cause

the strap to keep in the centre notwithstanding slight errors in fixing, &c.

When a rigger is intended to run loose on a shaft, the boss should be made very deep, not less than the width of the rim of the rigger in most cases, and bushed at both ends or throughout with gun-metal.

(91.) "*Weight of Riggers.*"—The weight of riggers varies very much with the proportions adopted and must be calculated in each case. But where the proportions are good, and the width varies with the diameter in the ratio $\sqrt[3]{D} \times 4 = b$, in which D = diameter in feet and b = breadth in inches, the weight may be approximately calculated by the rule $\frac{D^2 \times 30}{\sqrt{D}} = W$, in which D is the diameter in feet and W = the weight in lbs. Table 28 is calculated by this rule, which agrees very well with experience. For riggers of other widths and proportions of arms proper allowance must be made for such variations from the conditions assumed in the Table.

TABLE 28.—Of the WEIGHT of RIGGERS.

Diameter.		Width in Inches.	Weight.			Diameter.		Width in Inches.	Weight.		
ft.	in.		cwt.	qrs.	lbs.	ft.	in.		cwt.	qrs.	lbs.
1	0	4	0	1	6	3	0	$5\frac{3}{4}$	1	1	16
1	3	$4\frac{1}{4}$	0	1	14	3	6	$6\frac{1}{8}$	1	3	0
1	6	$4\frac{1}{2}$	0	1	27	4	0	$6\frac{3}{8}$	2	0	16
1	9	$4\frac{3}{4}$	0	2	14	4	6	$6\frac{5}{8}$	2	2	6
2	0	5	0	3	1	5	0	$6\frac{7}{8}$	3	0	0
2	3	$5\frac{1}{8}$	0	3	17	5	6	7	3	1	23
2	6	$5\frac{1}{4}$	1	0	6	6	0	$7\frac{1}{4}$	3	3	21
2	9	$5\frac{1}{2}$	1	0	25	7	0	$7\frac{5}{8}$	4	3	23

CHAPTER V.

ON THE PROPORTIONS OF KEYS FOR WHEELS AND RIGGERS.

(92.) There are five principal ways in which wheels, &c., may be secured to their places, and they are shown by Figs. 19–23. There are considerable differences in the relative efficiency and costliness of the different modes, and it must be left in most cases to the judgment of the engineer to select the one which will answer his purpose best.

Fig. 19 is a "hollow key," and is adapted only to turned shafts and bored wheels or riggers which have little work to do. The key is sunk into the boss of the rigger and is hollowed out to fit the shaft; it therefore drives only by friction, and is seldom used for anything but riggers with very light work. It has this advantage over other forms, that it allows the rigger to be easily shifted to another position on the shaft, and can be made secure at any point without any preparation of key-bed, &c.

Fig. 20 is a "flat key" and is capable of carrying considerably more power than the hollow key; a flat is filed on the shaft to receive it, and it is sunk in the boss of the rigger or wheel. This is the common mode of fixing all riggers except those of the largest size, and may also be used for small wheels that are not very heavily loaded.

Fig. 21 is the "sunk key," and this is the most useful and trustworthy form of key for round shafts where the strain is heavy, which is always the case with powerful wheels and riggers.

(93.) "*Proportions for Sunk Keys.*"—The sizes of sunk keys, and the depth to which they should be sunk in the shaft and in the boss of the wheel or rigger, are governed by the diameter of the shaft, but are not in simple proportion to the diameter. The following empirical rules are dictated by experience:— $\frac{D}{4} + \cdot 125 = B$; $\frac{D}{11} + \cdot 16 = T$; $\frac{D}{40} + \cdot 075 = d$; and T —

$d = d'$, in which D = the diameter of the shaft in inches, B = the breadth of the key in inches, T = the thickness of the key in inches, d = the depth sunk in the shaft, measured at the side of the key (see Fig. 1), and d' = the depth sunk in the boss of the wheel, also measured at the side of the key. Table 29

TABLE 29.—Of the PROPORTIONS of SUNK KEYS for WHEELS and RIGGERS.

Diameter of Shaft, inches	1	2	3	4	5	6
Breadth of Key .. "	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$	$1\frac{1}{8}$	$1\frac{3}{8}$	$1\frac{5}{8}$
Thickness of Key "	·25	·34	·43	·52	·61	·71
Depth sunk in Shaft "	·10	·125	·15	·175	·20	·225
Depth sunk in Wheel "	·15	·215	·28	·345	·41	·485

Diameter of Shaft, inches	7	8	9	10	11	12
Breadth of Key .. "	$1\frac{7}{8}$	$2\frac{1}{8}$	$2\frac{3}{8}$	$2\frac{5}{8}$	$2\frac{7}{8}$	$3\frac{1}{8}$
Thickness of Key "	·80	·89	·98	1·07	1·16	1·25
Depth sunk in Shaft "	·25	·275	·30	·325	·35	·375
Depth sunk in Wheel "	·55	·615	·68	·745	·81	·875

NOTE.—The depth sunk in the shaft and in the wheel is measured at the side of the key, and not at its centre, see (93) and Fig. 1; also the scales, Plate 4.

is calculated by these rules, and the scales in Plate 4 give the same particulars by direct measurement. The breadth of the key is equal from end to end, but the thickness must be regulated to give a uniform taper for the purpose of fitting tightly in its place. The amount of taper should be about $\frac{1}{8}$th inch to a foot in length, and the same must be given to the key-seat in the boss of the wheel and not to the key-bed in the shaft.

(94.) Fig. 22 shows the plan of hanging a wheel or rigger with four keys. It is commonly used for unturned shafts and unbored wheels. In this case there are four flat key-seats on the shaft; the keys may have the same proportions as in Table 29. The space between the shaft and the eye of the wheel allows for irregularities in casting, &c., and the exterior circumference of

the rigger, or pitch-circle of the wheel, may be adjusted to run truly by regulating the thickness of the keys. The same plan is frequently adopted with square shafts, as shown by the dotted lines at B in Fig. 22.

(95.) Fig. 23 gives the mode of fixing a wheel, &c., on a square shaft with eight keys. It is undoubtedly the most secure of all, and the most reliable where the strains are very great. In fitting a wheel on this principle four temporary keys are inserted at *a*, *b*, *c*, *d*, and by them the pitch-circle of the wheel is truly centred. The permanent keys are then accurately fitted with a taper of $\frac{1}{8}$th inch to a foot, &c., &c.

(96.) "*Key-boss for Strengthening the Key-way.*"—When a key-way is cut in the eye of a wheel or rigger, the boss is obviously weakened at that point, and a key-boss is necessary to restore the strength. The principles on which this should be done are not well understood by many practical millwrights. We sometimes see a key-boss like Fig 10, the length of B being little greater than the width of the key A; the result is that the key-boss is useless, for the line of fracture *a c* takes the shortest course, as shown by the Figure; and that course is obviously not lengthened by the boss B. The length of the course of fracture should be at least equal to the ordinary thickness of metal round the eye; in fact, it should be rather more, for it is well known that where there is a sharp corner, as at *c*, which may be regarded as an incipient crack, fracture is more likely to ensue than where no such angle is found. It is therefore advisable to make the shortest distance from *c*—say $\frac{1}{4}$ inch more than the thickness of metal round the eye. The key-bosses in the wheel and pinion (Fig. 7) are drawn on these principles, by taking a radius $\frac{1}{4}$ inch greater than the thickness of the boss and from the corner of the key, drawing the arcs *o p* and *q r*, and completing the key-boss by drawing the arc *p q* with the radius *p w*.

APPENDIX.

"*Contraction of Wheels in Casting.*"—The contraction which metals experience in cooling down from their melting-points to ordinary temperatures is very considerable, and allowance has to be made for it in fixing the size of the pattern. The contraction of straight bars, such as girders, &c., is tolerably regular, and in the case of cast iron it is very nearly $\frac{1}{8}$ inch to a foot; but with wheels it is not so much, and is found by experience to be irregular and anomalous. This is shown by the following Table from my 'Treatise on Heat.'

Extreme Diameter of Wheel-Casting.		Pitch in Inches.	Width of Teeth in Inches.	Contraction.		
				Total in Inches.	Per Foot.	
					Of Casting.	Of Pattern.
ft.	in.				inches.	inches.
10	$2\frac{3}{8}$	$3\frac{1}{2}$	12	1·08	·1059	·1040
6	$2\frac{9}{16}$	$3\frac{1}{4}$	9	·54	·0893	·0886
6	$1\frac{3}{8}$	$3\frac{1}{2}$	11	·375	·0613	·0610
5	$5\frac{9}{16}$	$3\frac{1}{2}$	11	·345	·0631	·0628
2	$11\frac{7}{8}$	$3\frac{1}{2}$	12	·11	·03896	·03884
2	$4\frac{15}{16}$	$3\frac{1}{4}$	9	·115	·0397	·0396

The irregularities in the *apparent* contraction arise in great part from the practice of "rapping" the pattern in the sand to make it an easy fit and enable it to be drawn out with facility. This is most influential in the case of small, heavy wheels of great width on the face; in some cases, and in rough hands, the casting of a small and wide pinion may be quite the full size of the pattern. The allowance to be made is therefore not uniform, but must be fixed by judgment; $\frac{1}{10}$ inch to a foot will do for wheels whose width on the face in inches is equal to their diameter in feet; when the width in inches is twice the diameter

in feet, say $\frac{1}{16}$ inch to a foot; when the width in inches is four times the diameter in feet, say $\frac{1}{25}$ inch to a foot, &c., &c. For the width, the allowance may be uniformly $\frac{1}{8}$ inch per foot.

"*Strength of Shafts, &c., for Screw-propellers.*"—The rules given in this work would apply with approximate correctness to propeller-shafts. For instance, in the case (3) where we had a pair of engines of 4000 nominal horse-power, by the rule (51) for common shafts we should require with 72 revolutions a wrought-iron shaft $\left(\frac{4000 \times 160}{72}\right) \sqrt[3]{} = 20\frac{5}{8}$ inches diameter; the real diameter, by Table 30, was $19\frac{3}{4}$ inches, and generally large propeller-shafts are somewhat smaller than that rule would give. There is some uncertainty in fixing the *nominal* horse-power of marine engines (see page 3), which has always to be *estimated* from the known gross indicated power. It will therefore be better to put the rule into another form, adapting it to the gross indicated power, and obtaining constants from propeller-shafts in practice.

The rule may take the form $\frac{M \times G}{R} = d^3$; in which G = gross indicated horse-power, including of course the friction, &c., of the engine and shaft, &c.; R = revolutions per minute; and M a constant obtained from satisfactory cases in practice by the rule $\frac{d^3 \times R}{G} = M$; its mean value may be taken at 65; its true value is given for several cases by Table 30. Taking

TABLE 30.—Of the DIAMETER of PROPELLER-SHAFTS in PRACTICE.

No.	Reputed Horse-power.	Observed Indicated Gross Horse-power.	Reduced Estimated Nominal Horse-power.	No. of Engine.	Revolutions per Minute.	Actual Diameter.	Calculated Diameter.	Value of M.	Makers.
						inches.	inches.		
1	1200	8000	4000	2	72	$19\frac{3}{4}$	$19\frac{3}{8}$	69	Penn.
2	700	4100	2050	2	58	16	$16\frac{5}{8}$	58	Humphreys.
3	400	1400	700	2	58	$11\frac{3}{8}$	$11\frac{5}{8}$	61	Rennie.

the mean value of M at 65 for two engines coupled at right angles, it would become $\frac{65 \times 1 \cdot 57}{1 \cdot 11} = 92$ for a single engine, and $\frac{65 \times 1 \cdot 05}{1 \cdot 11} = 61$ for three engines with cranks equally dividing the circle. See Theoretical Strength of Shafts, below.

"*Wheel-gearing to Screw-propellers.*"—Where wheels are used, their strength may be calculated by the ordinary rules in (37), &c., only it should be remembered that there being no fly-wheel to equalize the varying power of the pistons, the maximum strain should be taken as we have just shown to be necessary in the case of shafts. Thus, in the case No. 3 of Table 30, mortice-wheels, in the ratio of 2 to 1, were used, the large one being 14 feet diameter, 4 inches pitch, 48 inches wide on the face (in four sets of teeth, each 12 inches wide), 29 revolutions per minute. The mean *nominal* power was 700 horse; the maximum, therefore, would be $700 \times 1 \cdot 11 = 777$ nominal horse-power. By the rule in (37) the calculated power would be $\sqrt{14 \times 29} \times 16 \times 48 \times \cdot 05 = 773$ nominal horse-power.

"*On the Theoretical Strength of Shafts.*"—With a lever 1 foot radius, the mean torsional strength of a bar 1 inch diameter is 700 lbs. for wrought iron, and 450 lbs. for cast iron, breaking-weights. The safe working-strain may be taken at $\frac{1}{10}$th of the breaking-weight, and the circumference of a circle 1 foot radius being 6·28 feet, we have for wrought iron $70 \times 6 \cdot 28 = 440$ foot-pounds per revolution for a shaft 1 inch diameter.

In applying this to steam-engine shafts, it should be observed that with a single engine the maximum strain is 1·57 times the mean strain, see (51); with two engines, coupled at right angles, 1·11 to 1; and with three engines, 1·05 to 1. Taking the case of the engine in (3), we have 6000 nett indicated horse-power, which, with 72 revolutions, is equal to $\frac{6000 \times 33000}{72} = 2750000$ foot-pounds mean strain, or, with two engines, $2750000 \times 1 \cdot 11 = 3052500$ foot-pounds per minute, and we should require a

wrought-iron shaft $\left(\frac{3052500}{440}\right)\sqrt[3]{} = 19\frac{1}{8}$ inches diameter. The real diameter as made by Penn was $19\frac{3}{4}$ inches, as in Table 30. In the case No. 2 of that Table, the *nett* indicated power would be $2050 \times 1 \cdot 5 = 3075$ horse-power, and the shaft should be $\left(\frac{3075 \times 33000 \times 1 \cdot 11}{58 \times 440}\right)\sqrt[3]{} = 16\frac{3}{8}$ inches diameter. In the case No. 3 we have $700 \times 1 \cdot 5 = 1050$ *nett* indicated horse-power, and require a shaft $\left(\frac{1050 \times 33000 \times 1 \cdot 11}{58 \times 440}\right)\sqrt[3]{} = 11\frac{1}{2}$ inches diameter, &c. These sizes agree well with practice. See Table 30.

By the rules in (51) a common wrought-iron shaft, 1 inch diameter, and one revolution per minute, would be equal to $\frac{1^3 \times 1}{160} = \cdot 00625$ nominal horse-power; and as by (2) the nett indicated horse-power is $1 \cdot 5$ times the nominal, this is equal to $\cdot 00625 \times 1 \cdot 5 \times 33000 = 309$ foot-pounds per revolution; so that for common shafts, driving ordinary machinery, the safe working-strain is $\frac{309}{440 \times 10} = \cdot 07$, or $\frac{1}{14}$th of the breaking or crippling strain; but with screw-propeller shafts, and in other cases where the work is regular, the working-strain may be as much as $\frac{1}{10}$th of the breaking-strain.

Professor Rankine has recently shown that the weight of the shaft itself creates a strain, which so far coincides with that produced by torsion, as to require in ordinary cases about 11 per cent. extra strength in the shaft, and with that addition he calculates that a shaft carrying 5500 nett indicated horse-power, with fifty-four revolutions, should be $17 \cdot 21$ inches diameter. But he takes the working strain at $\frac{1}{5}$th to $\frac{1}{6}$th the breaking weight, which is certainly too high: by the rules just given we have $\left(\frac{5500 \times 33000 \times 1 \cdot 11}{54 \times 440}\right)\sqrt[3]{} = 20$ inches diameter for the torsional strain alone.

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Plate 1

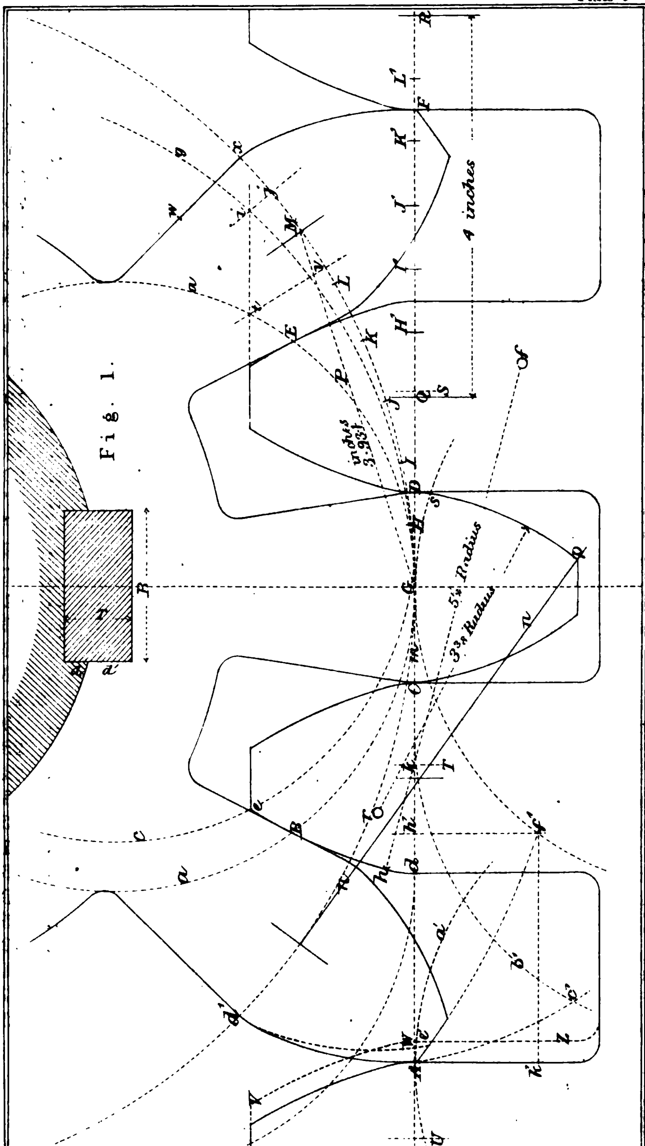

Fig. 1.

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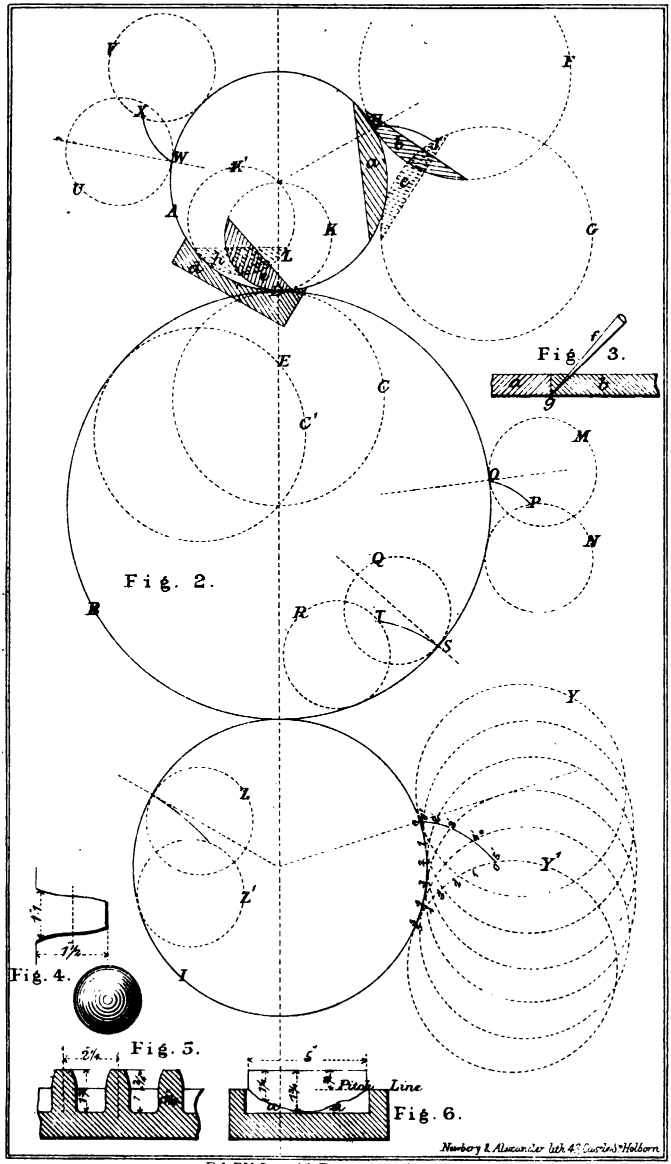

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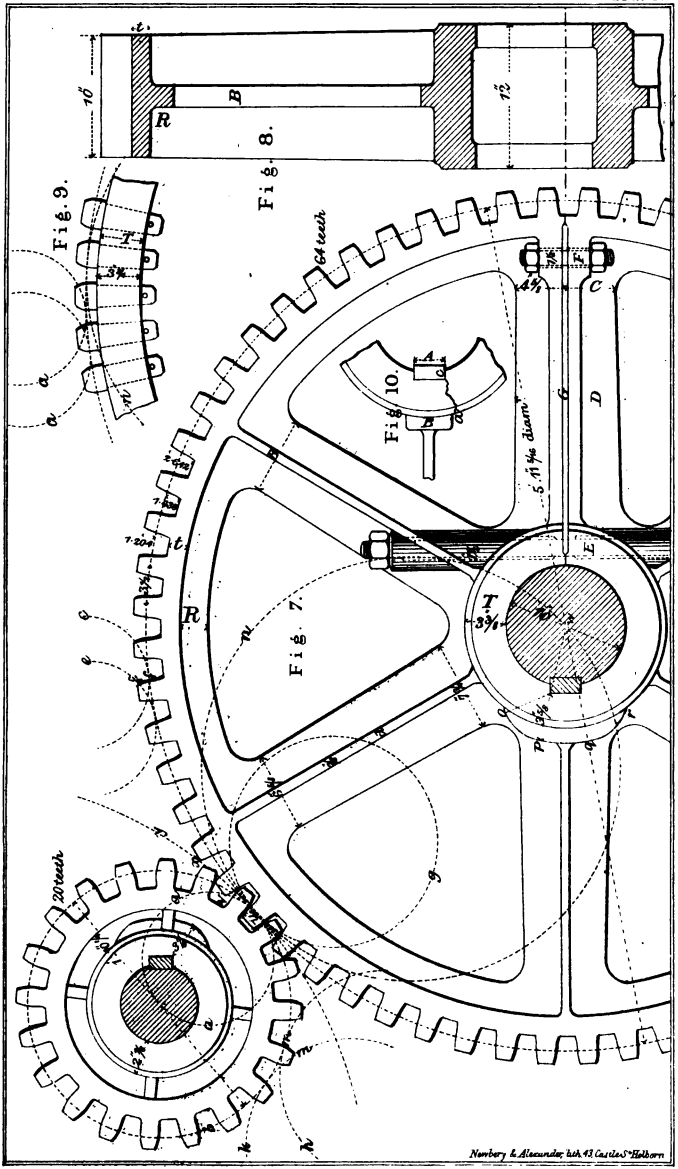

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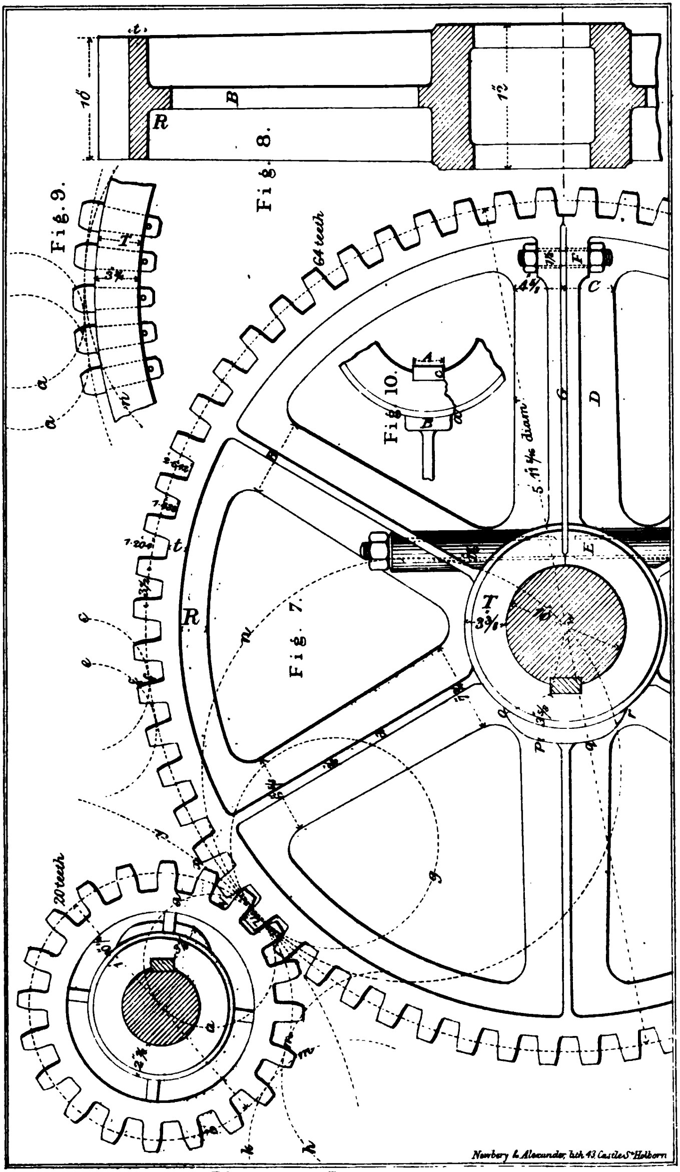

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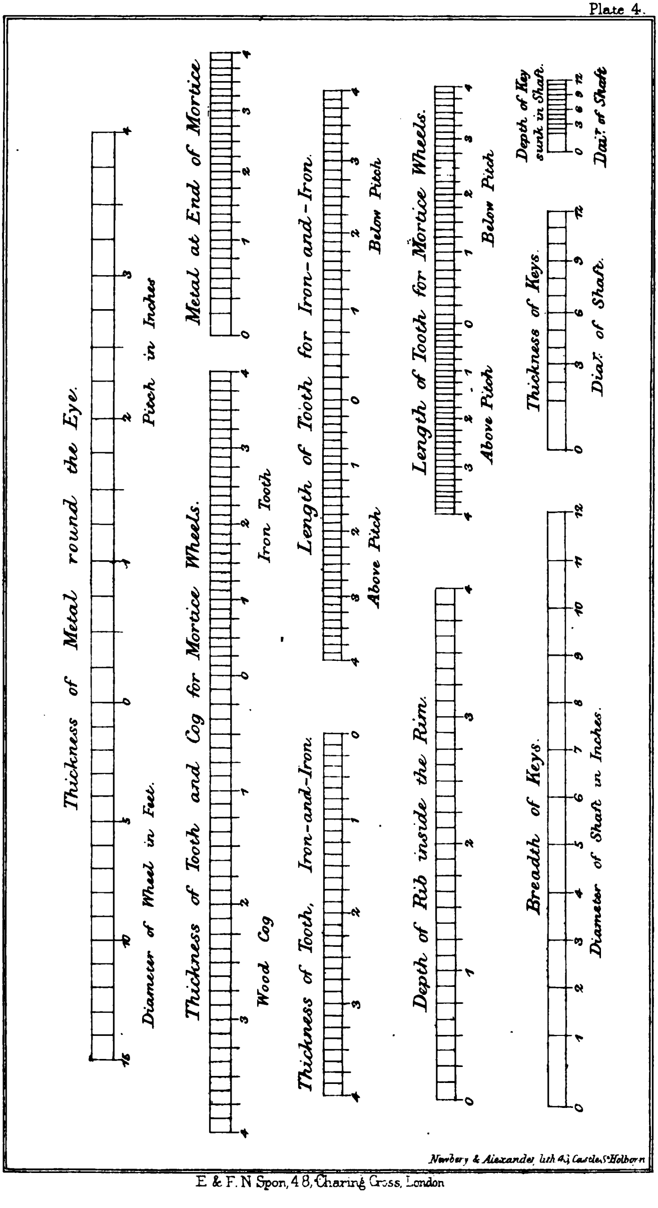

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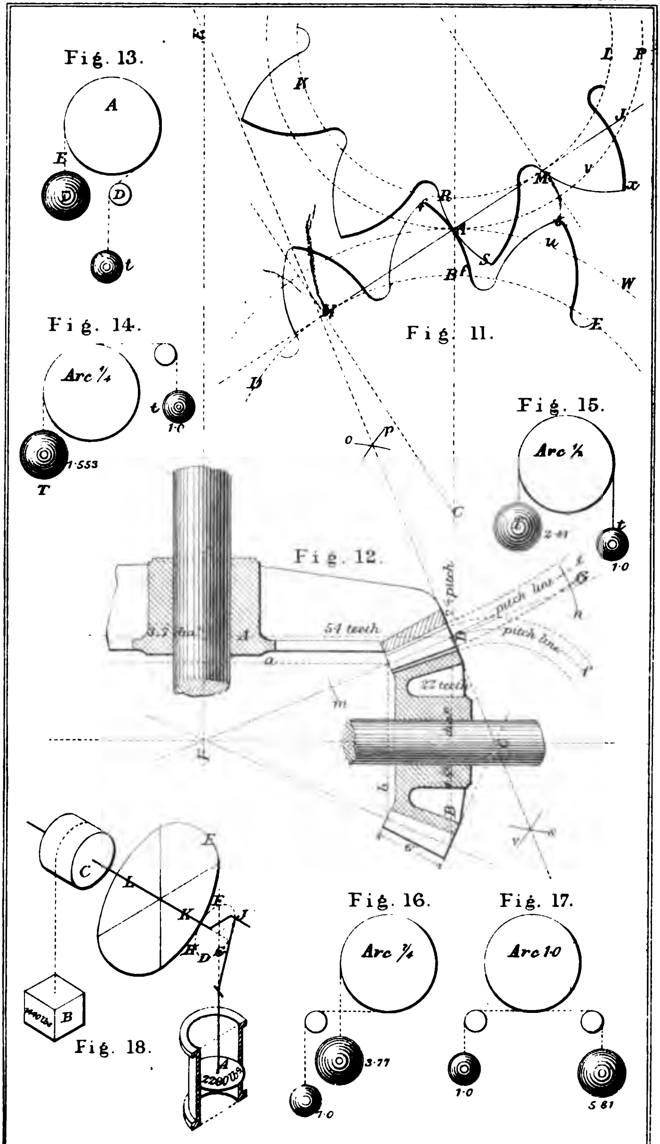

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Plate 6.

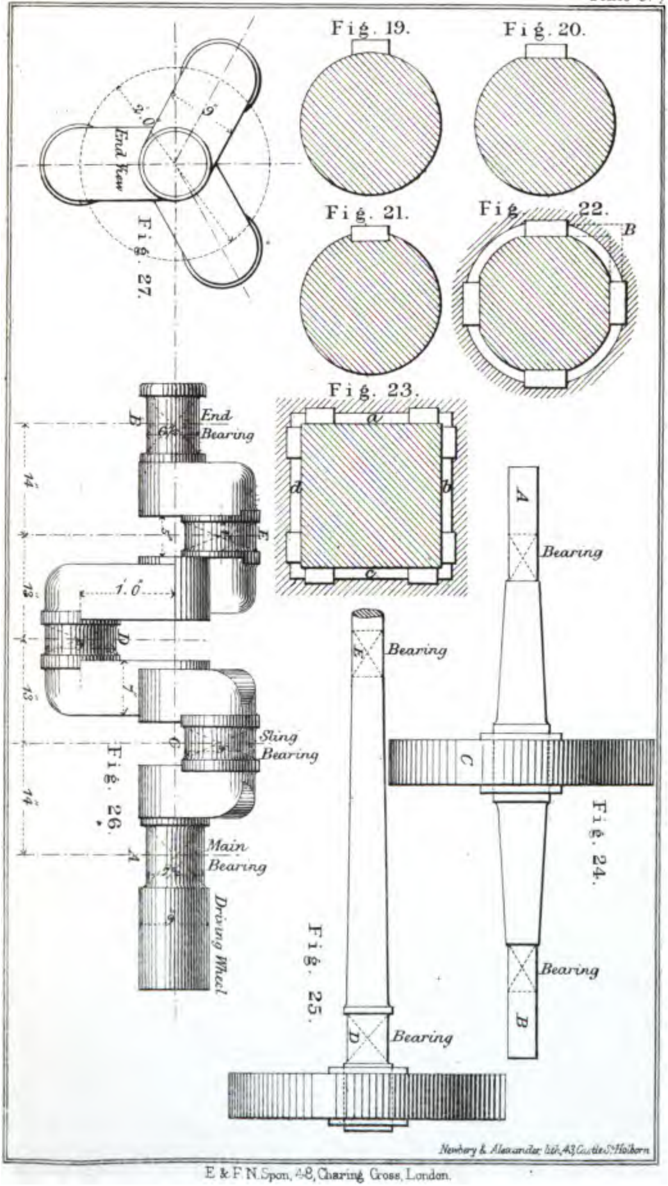

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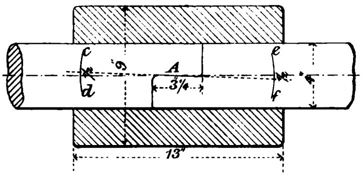
Fig. 28.



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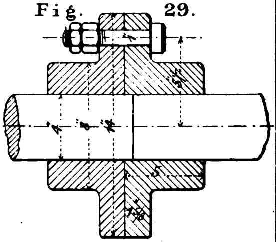
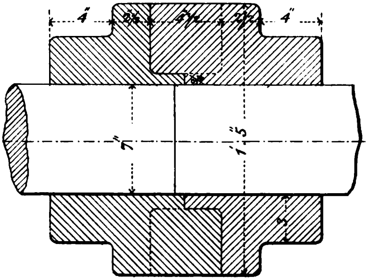


Fig. 30.



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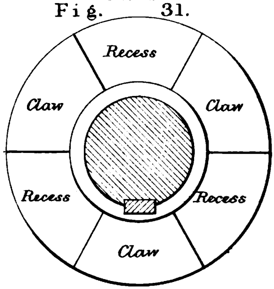
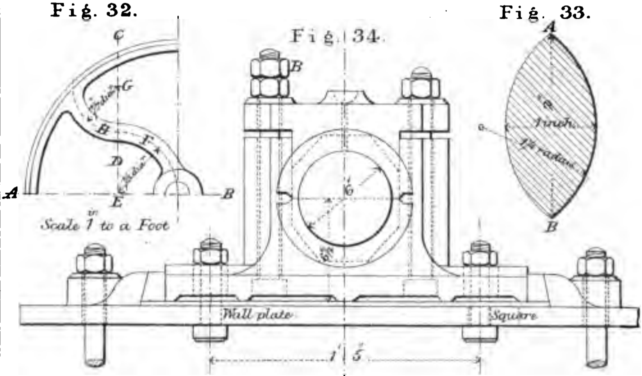


Fig. 32.

Fig. 33.

Fig. 34.



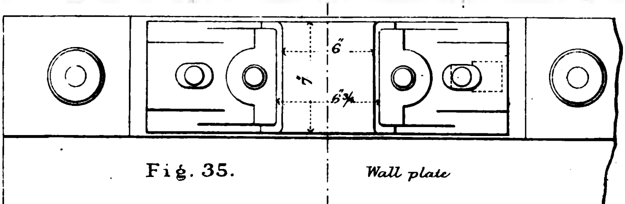


Fig. 35.

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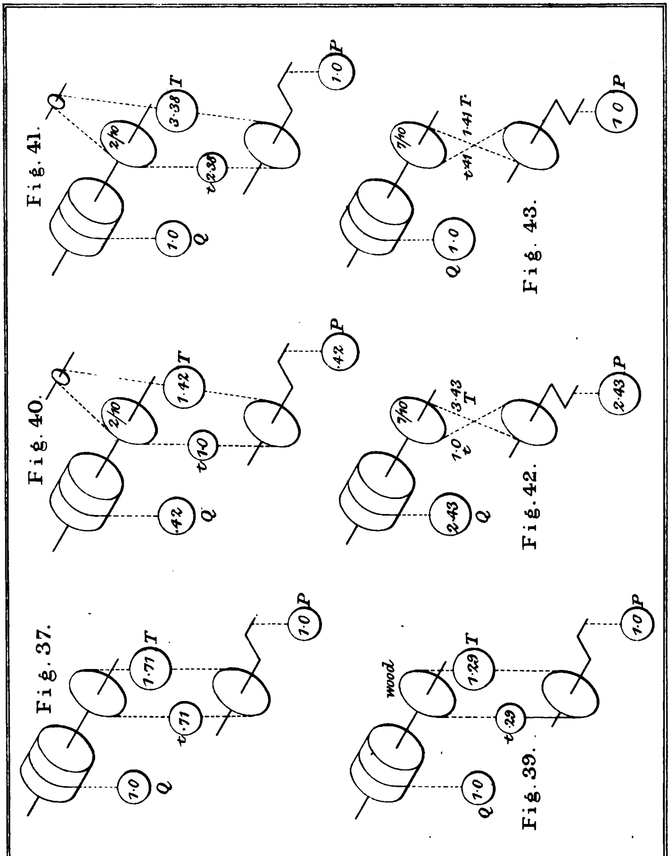

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